BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 2, October 1984

REDUCIBILITY OF STANDARD REPRESENTATIONS

BY DAN BARBASCH[1] AND DAVID A. VOGAN, JR.[2]

Let G be a real linear reductive group with abelian Cartan subgroups. Unexplained notation, in general, follows [**3** and **6**]. Fix a parabolic subgroup $P = MAN$ of G and a representation δ of M in the limits of the discrete series. The continuous family of representations

$$\pi(\nu) = \mathrm{Ind}_P^G(\delta \otimes \nu \otimes 1) \qquad (\nu \in \hat{A} \cong \mathfrak{a}^*)$$

is a typical series of *standard representations* of G. (These are not, in general, unitary since ν may not be a unitary character of A.) In order to apply certain "continuity arguments" in the study of unitary representations of G, it is necessary to know for which values of ν the representations $\pi(\nu)$ is reducible. We sketch here an explicit answer to this question for classical groups. (Our techniques reduce the problem for exceptional groups to a (long) finite calculation.) The continuity arguments mentioned above require a similar understanding of reducibility for some larger class (it is not yet clear *what* larger class) of induced representations. Some of our techniques also apply to this more general problem.

Write $\overline{\pi}(\nu)$ for the direct sum of the Langlands subquotients of $\pi(\nu)$. These are the irreducible composition factors of $\pi(\nu)$ whose matrix coefficients exhibit the largest possible growth at infinity [**1**]. (Alternatively [**4**], they may be characterized by the fact that their restrictions to a maximal compact subgroup contain representations which are as small as possible.) Obviously $\pi(\nu)$ is reducible if and only if at least one of the following conditions holds: $\overline{\pi}(\nu)$ is reducible; or $\pi(\nu)$ has some composition factor not in $\overline{\pi}(\nu)$. We write the second possiblity as $\pi(\nu) \neq \overline{\pi}(\nu)$. Now Knapp and Zuckerman have determined in [**2**] exactly when the first possibility occurs: ν must belong to one of finitely many linear subspaces in $\mathfrak{a}^*$, which are explicitly described in terms of the inducing representation δ. We must therefore explain when $\pi(\nu) \neq \overline{\pi}(\nu)$.

In writing a Langlands decomposition $P = MAN$, we have implicitly fixed a Cartan involution θ. Choose a θ-stable compact Cartan subgroup $T \subseteq M$ and write $H = TA$ for the corresponding θ-stable Cartan subgroup of G. The representation δ determines (up to conjugacy under $W(M,T)$) a positive root system $\Delta^+(\mathfrak{m}, \mathfrak{t})$ and a Harish-Chandra parameter $\lambda \in \mathfrak{t}^*$. Put

$$\overline{\gamma} = (\lambda, \nu) \in \mathfrak{t}^* + \mathfrak{a}^* \cong \mathfrak{h}^*,$$

$$R(\delta \otimes \nu) = \{\alpha \in \Delta(\mathfrak{g}, \mathfrak{h}) | \langle \check{\alpha}, \overline{\gamma} \rangle \in \mathbf{Z}\};$$

as usual, $\check{\alpha}$ denotes the coroot $2\alpha/\langle \alpha, \alpha \rangle$.

Received by the editors May 18, 1984 and, in revised form, July 12, 1984.
1980 *Mathematics Subject Classification.* Primary 22E46.
[1]Supported by an NSF grant and a Rutgers Research Council grant.
[2]Supported in part by NSF grant MCS-8202127.