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COMPACT LIE GROUPS ASSOCIATED WITH ENDOMORPHISMS OF C^*-ALGEBRAS

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The work described in this announcement is motivated by a long-standing problem in quantum field theory. Experimental evidence and theoretical considerations [**1, 2**] suggest that superselection structure is determined by the representation theory of a compact group, the gauge group of the first kind. Attempts to prove the existence of this group from the general principles of quantum field theory pinpoint the inadequacy of the classical Tannaka-Krein duality theorem for compact groups: it enables one to recognize the representation theory only when the intertwining operators are given concretely as linear operators between representation spaces. The discussion of superselection theory in algebraic quantum field theory leads not to representations of a compact group but to endomorphisms of a C^*-algebra, the C^*-algebra of local observables, and the intertwining operators intertwine these endomorphisms [**3**]. We announce here, in the setting of C^*-algebras, the basic results which have allowed us to resolve this problem.

We give conditions on an endomorphism ρ of a C^*-algebra $\mathcal{A}$ with unit and trivial centre in terms of intertwining operators for powers of this endomorphism which suffice to determine a compact Lie group G and an action of a G-dual on $\mathcal{A}$. It is convenient to begin by describing the C^*-systems $\{\mathcal{B}, G, \alpha\}$ which arise if we take a cross product of $\mathcal{A}$ by the action of the G-dual.

Thus, we consider a C^*-algebra $\mathcal{B}$ carrying a faithful continuous action α of a compact group G by automorphisms and a Hilbert space $H \subset \mathcal{B}$ of dimension d, $1 < d < +\infty$, i.e. we have an orthonornal basis $\psi_1, \psi_2, \ldots, \psi_d$ of H consisting of isometries satisfying

$$\psi_i^* \psi_j = \delta_{ij} I, \qquad \sum_{i=1}^{d} \psi_i \psi_i^* = I.$$

Let $\mathcal{B}^\alpha$ denote the subalgebra of the fixed points. We suppose:
(a) H and $\mathcal{B}^\alpha$ generate $\mathcal{B}$;
(b) $\alpha_g(H) = H$, $g \in G$;
(c) $\operatorname{Det} \alpha_g | H = 1$, $g \in G$;
(d) $(\mathcal{B}^\alpha)' \cap \mathcal{B} = \mathbf{C}I$.

The first three conditions imply that G is isomorphic to a closed subgroup of $\mathrm{SU}(d)$ and, hence, is a Lie group. The final condition leads to a particularly simple class of cross products.

We first give some examples of this situation.

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