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TOEPLITZ OPERATORS AND SOLVABLE C^*-ALGEBRAS ON HERMITIAN SYMMETRIC SPACES

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Bounded symmetric domains (Cartan domains and exceptional domains) are higher-dimensional generalizations of the open unit disc. In this note we give a structure theory for the C^*-algebra $\mathcal{T}$ generated by all *Toeplitz operators* $T_f(h) := P(fh)$ with continuous symbol function $f \in \mathcal{C}(S)$ on the Shilov boundary S of a bounded symmetric domain D of arbitrary rank r. Here h belongs to the *Hardy space* $H^2(S)$, and $P\colon L^2(S) \to H^2(S)$ is the Szegö projection. For domains of rank 1 and tube domains of rank 2, the structure of $\mathcal{T}$ has been determined in [**1, 2**]. In these cases Toeplitz operators are closely related to pseudodifferential operators. For the open unit disc, $\mathcal{T}$ is the C^*-algebra generated by the unilateral shift.

The structure theory for the general case [**12**] is based on the fact that D can be realized as the open unit ball of a unique *Jordan triple system* Z [**7**, Theorem 4.1]. Denoting the Jordan triple product by $\{uv^*w\}$, a *tripotent* $e \in Z$ satisfies $\{ee^*e\} = e$. Tripotents generalize the partial isometries of matrix algebras and determine the *boundary structure* of $D \subset Z$ (cf. [**7**, Theorem 6.3]). Our principal result ([**12**]; cf. also [**3, 4, 8**]) is the following:

THEOREM 1. *The Toeplitz C^*-algebra $\mathcal{T}$ associated with a bounded symmetric domain $D \subset Z$ of rank r is solvable of length r, i.e. there exists a chain*

$$\{0\} = \mathcal{I}_0 \subset \mathcal{I}_1 \subset \mathcal{I}_2 \subset \cdots \subset \mathcal{I}_r \subset \mathcal{I}_{r+1} = \mathcal{T}$$

of closed two-sided ideals $\mathcal{I}_k$ such that for $0 \leq k \leq r$ there is a C^*-algebra isomorphism ("k-symbol")

$$\sigma_k\colon \mathcal{I}_{k+1}/\mathcal{I}_k \to \mathcal{C}(S_k) \otimes \mathcal{K}(H_k),$$

where S_k denotes the compact manifold of all tripotents $e \in Z$ of rank k and $\mathcal{K}(H_k)$ denotes the C^*-algebra of all compact operators on a Hilbert space H_k. Further, $\dim(H_k) = \infty$ *for $k < r$ and* $\dim(H_r) = 1$.

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