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RIEMANN SURFACES AND THEIR ASSOCIATED WIRTINGER VARIETIES

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1. The canonical imbedding of a compact Riemann surface in its associated Jacobi variety, an algebraic complex torus, has long been of fundamental importance in the study of Riemann surfaces, and the basic holomorphic functions that arise in the study of algebraic tori, the theta functions, naturally play an essential role in this part of the theory. The central role and essential properties of the simple theta function in the study of Riemann surfaces were established by Riemann and lie at the core of the subject. Some of the basic properties of higher-order theta functions on general algebraic tori, principally those properties involved in the projective imbedding of these tori, were established by Kummer for two-dimensional tori and extended by Wirtinger to higher-dimensional tori, and investigations in this direction have been pursued fairly steadily ever since. Some quite deep and subtile properties of the higher-order theta functions of Jacobi varieties, as distinct from general algebraic tori, were established by Frobenius and Schottky, and investigations in this direction have also been pursued ever since, perhaps somewhat fitfully and implicitly at the beginning (since the higher-order theta functions can be expressed as various products of translates of simple theta functions and thereby disguised, sometimes conveniently and sometimes confusingly) but with increasing vigor, particularly in the last decade. Part of this current activity, particularly among algebraic geometers, derives from Fay's stimulating book on theta functions and Riemann surfaces, while another part no doubt derives from the role that theta functions of Jacobi varieties have been discovered to play in providing explicit solutions of some important nonlinear partial differential equations. Both of these inspirations have led to fascinating new approaches to and results about Schottky's problem of characterizing Jacobi varieties among more general algebraic tori; there has thus been a surprising revival of interest in this once obscure but always intriguing topic.

The aim of the present paper is to provide a survey of some of this recent activity that most directly involves the study of Riemann surfaces itself, together with a discussion of a few new results in the directions surveyed. The

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