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Theory of charges, a study of finitely additive measures, by K. P. S. Bhaskara Rao and M. Bhaskara Rao, Academic Press, London, 1983, x + 315 pp., $55.00. ISBN 0-1209-5780-9

A charge is a finitely additive, extended real-valued set function defined on a field of sets. The notion is thus a familiar one even to those who may not have used the term. But why should we study finitely additive measures? Haven't Borel and Lebesgue made them obsolete? We have become so accustomed to countable additivity that most of us take it for granted and feel we would be lost without it. Nevertheless, no less an authority than S. Bochner is quoted as having remarked that finitely additive measures are more interesting, more difficult to handle, and perhaps more important than countably additive ones.

Everyone knows that density is a natural measure in the set of positive integers, and that it has proved very useful in number theory despite the fact that it is only finitely additive. Sometimes density is linked to a countably additive measure. For example, under an ergodic transformation of a normalized measure space, almost all points generate sequences of images that fall in any given measurable set with a frequency (that is, density) equal to the measure of the set. The law of large numbers establishes a similar link between a countably additive probability and densities on almost all sample sequences.

If countable additivity were really indispensable one might wonder how mathematicians managed to get along without it for so long. Of course, length, area and volume are actually countably additive, although this fact was not fully appreciated or exploited until the end of the last century. There are other circumstances in which countable additivity comes as a bonus; for example, when the domain is the field of closed open subsets of a compact space. As a consequence, any charge can be represented by a countably additive charge on a corresponding Stone space, but this representation is too esoteric to be of much use except for special purposes.

It is a remarkable fact that countable additivity is sometimes forced by an invariance requirement. D. Sullivan and G. A. Margulis have recently shown that, for $n \geqslant 3$, Lebesgue measure is the only finitely additive measure on the bounded measurable subsets of R^n that normalizes the unit cube and is isometry-invariant, thus settling a very old and classical problem of Ruziewicz.