BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 11, Number 1, July 1984

ASYMPTOTIC PROPERTIES OF NORMAL AND NONNORMAL HOLOMORPHIC MAPPINGS

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Let Ω be a bounded domain in $\mathbf{C}^m$ with Kobayashi metric k_Ω and the corresponding differential metric K_Ω. Let N be a hermitian manifold with hermitian inner product h_N and hermitian metric d_N. Let $\mathcal{H}(\Omega, N)$ denote the class of all holomorphic mappings $f: \Omega \to N$.

In this paper, generalizing the classical notions of normal functions [7], Bloch functions [1], regular sequences [9], and P-point sequences [3] of one complex variable to holomorphic mappings in $\mathcal{H}(\Omega, N)$ we obtain certain relations existing between these notions. (These notions have been generalized by several authors. See [4, 10, 11] for example.) These results are then used to draw some interesting conclusions on the boundary behavior of normal and nonnormal holomorphic mappings.

The context of this paper and other related results with complete proofs will be published elsewhere [5].

We shall say that $f \in \mathcal{H}(\Omega, N)$ is: *normal* if Ω is homogeneous, i.e., the group of holomorphic automorphisms $\mathrm{Aut}(\Omega)$ is transitive, and the family $\{f \circ \varphi \colon \varphi \in \mathrm{Aut}(\Omega)\}$ forms a normal family (in the sense of H. Wu [12]); *Bloch* if

$$Qf \equiv \sup\{(Qf)(z) : z \in \Omega\} < \infty, \tag{1a}$$

where

$$(Qf)(z) = \sup_{|\xi|=1} \frac{h_N(f(z), df(z)\xi)}{K_\Omega(z,\xi)} \qquad (z \in \Omega, \xi \in \mathbf{C}^m). \tag{1b}$$

Denote by $\mathcal{N}(\Omega, N)$ and $\mathcal{B}(\Omega, N)$ the classes of normal and Bloch mappings $f: \Omega \to N$, respectively.

THEOREM 1. *Let Ω be a homogeneous bounded domain and N a complete hermitian manifold. Then $\mathcal{B}(\Omega, N)$ is a proper subset of $\mathcal{N}(\Omega, N)$. If, in particular, N is compact, then $\mathcal{B}(\Omega, N) = \mathcal{N}(\Omega, N)$. Moreover, if Ω is symmetric and N is negatively curved, i.e., the holomorphic sectional curvature of N is bounded above by a negative constant, then $\mathcal{B}(\Omega, N) = \mathcal{H}(\Omega, N)$. In the opposite direction, if Ω is strongly pseudoconvex and $N = \mathbf{C}^n$, then $\mathcal{B}(\Omega, \mathbf{C}^n)$ contains all bounded holomorphic mappings properly.*

Following Seidel and Walsh [9] and Gauthier [3], we shall define a sequence $\{p_n\}$ of points in Ω to be: *regular* for $f \in \mathcal{H}(\Omega, N)$ if there exists a $\delta > 0$ such

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Received by the editors May 16, 1983 and, in revised form, June 7, 1983.
1980 *Mathematics Subject Classification*. Primary 32H25, 32A40; Secondary 30D45.
[1]This work was supported in part by NSF MCS80-02915.

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0273-0979/84 $1.00 + $.25 per page