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SINGULAR MINIMIZERS FOR REGULAR ONE-DIMENSIONAL PROBLEMS IN THE CALCULUS OF VARIATIONS

BY JOHN M. BALL AND VICTOR J. MIZEL

We announce some surprising examples of regular one-dimensional problems of the calculus of variations possessing singular absolute minimizers. These minimizers *do not satisfy the usual integrated version of the Euler-Lagrange equation.* The examples all concern integrals of the form

$$I(u) = \int_a^b f(x, u(x), u'(x))\, dx,$$

where $[a, b]$ is a finite interval, $'$ denotes d/dx, $f = f(x, u, p)$ is C^∞, $f \geq 0$ and $f_{pp} > 0$ (regularity). We consider the problem of minimizing I in the set $\mathcal{A}$ of absolutely continuous functions $u\colon [a, b] \to \mathbf{R}$ satisfying $u(a) = \alpha$, $u(b) = \beta$, where α and β are given constants. As is well known, if a minimizer u of I in $\mathcal{A}$ is Lipschitz continuous then u is smooth and satisfies the Euler-Lagrange equation

$$\text{(EL)} \qquad (d/dx) f_p = f_u,$$

and the DuBois-Reymond equation

$$\text{(DBR)} \qquad (d/dx)(f - u' f_p) = f_x,$$

for all $x \in [a, b]$. A little-known partial regularity theorem of Tonelli [**1923**, Vol. 2, p. 359] asserts that given any minimizer u of I in $\mathcal{A}$, (i) there exists a closed set $E \subset [a, b]$ of measure zero such that $u \in C^\infty([a, b]\backslash E)$ and

$$\lim_{\substack{\operatorname{dist}(x,E)\to 0\\ x\notin E}} |u'(x)| = \infty,$$

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