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Schrödinger-type operators with continuous spectra, by M. S. P. Eastham and H. Kalf, Research Notes in Mathematics, Vol. 65, Pitman Advanced Publishing Program, Boston, 1982, 281 pp., $24.95. ISBN 0-2730-8526-3

To a mathematical physicist the title *Schrödinger-type operators with continuous spectra* arouses expectations of a wide variety of topics connected with quantum dynamics, so it is something of a surprise to open this book and find no reference to wave operators or the other paraphernalia of scattering theory. Eastham and Kalf soon make it clear, however, that an accurate subtitle might have been *A monograph on the possibility of eigenvalues embedded in the continuum*. The prospective reader should be aware of this narrower scope.

The spectral theorem assigns a unique spectral family, or projection-valued measure on the real line, to any selfadjoint operator on Hilbert space. Von Neumann **[12]** placed the spectral theorem at the heart of his axiomatic formulation of quantum mechanics, and as a result spectral analysis of Schrödinger operators

$$-\Delta + q(x), \tag{1}$$

defined on appropriate subspaces of $L^2(\mathbf{R}^n)$, is a central part of mathematical physics. There are two useful ways to decompose the spectrum of a selfadjoint operator. One of them divides σ_{d}, consisting of discrete eigenvalues of finite multiplicity, from σ_{e}, the essential spectrum, consisting of everything else. The utility of this arises from Weyl's well-known theorems on the invariance of σ_{e} under compact perturbations or, in the case of Sturm-Liouville operators, changes in boundary conditions **[19]**. The second, more measure-theoretic division distinguishes among σ_{p}, the point spectrum, consisting of eigenvalues whether isolated or not; σ_{ac}, the absolutely continuous spectrum, which is associated with the part of the spectral family orthogonal to the eigenvectors and in a sense absolutely continuous with respect to Lebesgue measure; and σ_{sc}, the singular continuous spectrum. The continuous spectrum σ_{c} is the union of σ_{ac} and σ_{sc}. The definition allows the possibility that σ_{c} and σ_{p} intersect. In one-body physics, or two-body physics after removal of the center of mass, the potential q tends to zero at infinity, and typically

$$\sigma_{\mathrm{e}} = \sigma_{\mathrm{c}} = \sigma_{\mathrm{ac}} = [0, \infty), \tag{2}$$

$$\sigma_{\mathrm{sc}} = \varnothing, \tag{3}$$

$$\sigma_p \cap \sigma_c = \varnothing \text{ (or, more rarely, } \{0\}). \tag{4}$$

This, at least, is expected on physical grounds, and there is a significant industry manufacturing theorems guaranteeing (2)–(4), as well as examples where they are false, contrary to naive expectation. This book is concerned with (4). The two ways of dividing up the spectrum go back to the early years