BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 10, Number 2, April 1984

ON THE RELATIONS BETWEEN CHARACTERISTIC CLASSES OF STABLE BUNDLES OF RANK 2 OVER AN ALGEBRAIC CURVE

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ABSTRACT. We describe a complete set of generators and relations for a certain quotient of the rational cohomology ring of the moduli space of stable bundles of rank 2 and fixed determinant of odd degree over a nonsingular complex algebraic curve. The formulae for the relations apply in any genus and are relatively simple.

1. Introduction. Let $S = U_L(2,1)$ denote the moduli space of stable bundles of rank 2 and determinant L of degree 1 over a nonsingular complete algebraic curve X of genus $g \geq 2$ defined over the complex numbers. The Betti numbers of S were determined some time ago in [**3**], and generators for $H^*(S;Q)$ were given in [**4**]. Recently there has been renewed interest in obtaining a complete description of $H^*(S;Q)$, particularly in connection with the work of M. F. Atiyah and R. Bott [**1**, §9]. David Mumford and Dave Bayer have performed some calculations on a computer, which provide evidence in support of some conjectures of Mumford. In this note we use the topological methods of [**3, 4**] to obtain some information about relations in $H^*(S;Q)$; these provide further support for Mumford's conjectures.

2. The main theorem. We recall the generators for $H^*(S;Q)$ given in [**4**], namely $\alpha \in H^2(S;Z)$; $\psi_1, \ldots, \psi_{2g} \in H^3(S;Z)$; $\beta \in H^4(S;Z)$. A little care is needed over the definition of the ψ_i. We first choose a *symplectic* basis $a_1, \ldots, a_{2g}$ for $H^1(X;Z)$ (with respect to the skew-symmetric form given by Poincaré duality); then the ψ_i are defined by the equation

$$\psi = \psi_1 \otimes a_1 + \cdots + \psi_{2g} \otimes a_{2g},$$

where ψ is the component in $H^3(S;Z) \otimes H^1(X;Z)$ of the second Chern class of a universal bundle on $S \times X$. We write

$$\sigma = \psi_1\psi_2 + \cdots + \psi_{2g-1}\psi_{2g} \in H^6(X;Z),$$

so that $\psi^2[X] = 2\sigma$.

THEOREM 1. *Let A denote the ring $H^*(S;Q)/\langle\beta\rangle$. Then the monomials*

$$(1) \qquad \alpha^s\psi_{q_1}\cdots\psi_{q_t} \qquad (s,t \geq 0,\ 1 \leq q_1 < q_2 < \cdots < q_t \leq 2g,\ s+t < g)$$

form a basis for A as a vector space over Q. Moroever, whenever $s+t \geq g$,

$$(2) \qquad [\alpha^s + f_s(\alpha,\sigma)]\psi_{q_1}\cdots\psi_{q_t} = 0,$$

Received by the editors September 6, 1983.
1980 *Mathematics Subject Classification*. Primary 14D22, 14F05, 14F25; Secondary 55R40.