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LOCAL RINGS OF FINITE SIMPLICIAL DIMENSION

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In this note R denotes a (noetherian, commutative) local ring with residue field k. Our purpose is to determine those R, over which k has finite (co-)homological dimension *as an R-algebra* in the simplicial theory of André [**1**] and Quillen [**11**]. Recall that regular local rings are characterized in this theory by the vanishing of the homology group $D_2(k|R)$. Furthermore, it is known that each of the conditions (i) $D_3(k|R) = 0$, (ii) $D_4(k|R) = 0$, (iii) $D_q(k|R) = 0$ for $q \geq 3$, is equivalent to R being a complete intersection, by which we mean that in some (hence in any) Cohen presentation of the completion $\hat{R}$ as a homomorphic image of a regular local ring $\tilde{R}$, the ideal $\operatorname{Ker}(\tilde{R} \to \hat{R})$ is generated by an $\tilde{R}$-regular sequence.

THEOREM 1. *If $D_q(k|R) = 0$ for q sufficiently large, then R is a local complete intersection.*

REMARK 1. The previous statement proves a conjecture of Quillen [**11**, Conjecture 11.7] and answers a question of André [**1**, p. 118]. When $\operatorname{char}(k) = 0$, its validity is established by [**11**, Theorem 7.3] and Gulliksen's result in [**10**].

REMARK 2. It has been shown by the author and Halperin [**4**] that in characteristic zero the conclusion of the theorem holds under the (much) weaker assumption that $D_q(k|R) = 0$ for *infinitely many* values of q. It is not known whether the restriction is essential, and in fact it is an open question, in any characteristic, whether the cotangent complex is rigid, i.e.: Does $D_q(k|R) = 0$ for *a single* $q \geq 1$ imply that R is a complete intersection?

The proof of Theorem 1 uses some precise information on the growth of the coefficients of the formal power series $P_R(t) = \sum_{j\geq 0} \dim_k \operatorname{Tor}_j^R(k,k)t^j$. For our present purpose it is best expressed in terms of the radius of convergence $r(P_R(t))$. Note that the inequality $r(P_R(t)) > 0$ has been known for a long time to hold for any local ring R, and that for complete intersections one even has $r(P_R(t)) \geq 1$.

THEOREM 2. *The inequality $r(P_R(t)) \geq 1$ characterizes complete intersections.*

REMARK 3. The last result has been conjectured both by Golod and by Gulliksen, and proved, in case $R = \bigoplus_{i\geq 0} R_i$ is graded with $R_0 = k$ a field of characteristic zero, by Felix and Thomas [**9**]. Results related to Theorem 2 are discussed in [**2**]; complete proofs will appear in [**3**].

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