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DELIGNE HOMOLOGY AND ABEL-JACOBI MAPS

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The purpose of this note is to announce a functorial description of the Abel-Jacobi homomorphisms of [7] by means of a theory of cycle classes taking values in Deligne homology, which is part of a Poincaré duality theory satisfying the axioms of [3] on the category of all schemes of finite type over **C**. One consequence of this approach is that the cycle class map is an edge homomorphism in the coniveau spectral sequence, and so conjectures on the structure of the Chow groups, such as Bloch's conjecture [2], can be interpreted in terms of the vanishing (or not) of differentials in this spectral sequence. This formalism, because of its distinction between homology and cohomology, may also give a good framework for studying specialization questions. Deligne cohomology was originally defined by Deligne for X a proper smooth variety (or manifold) over **C**:

$$H^p(X, D(q)) = \mathbf{H}^p(X^{an}, \mathbf{Z} \xrightarrow{(2\pi i)^q} \mathcal{O}_X \to \cdots \to \Omega_X^{q-1}),$$

so there are exact sequences for $p \geq 0$:

$$0 \to J^p(X) \to H^{2p}(X, D(p)) \to Hge^p(X) \to 0$$

($J^p = p$th Griffith's intermediate Jacobian [7]; $Hge^p(X)$ is the group of integral (p,p) cohomology classes). A description of these groups may be found in [5] together with a construction of cycle classes in them. The extension of the theory to noncompact, possibly singular, varieties, or even simplicial schemes, was obtained independently (and probably first) by A. A. Beilinson [1]. The definition of Deligne homology was inspired by El-Zein's and Zucker's use of currents to prove the covariant functoriality of the Abel-Jacobi maps in [5]. J. King has also proved naturality of the Abel-Jacobi maps by more explicit methods [8]. However, neither [5] nor [8] describe theories with the full range of formal properties described here.

Let $\mathcal{V}$ be the category of schemes of finite type over **C**, $\mathcal{V}^*$ the category of closed immersions $Y \to X$ in $\mathcal{V}$ with morphisms Cartesian squares as in [3], and $\mathcal{V}_*$ the category of proper morphisms in $\mathcal{V}$.

THEOREM 1. *There is a Poincaré duality theory with supports on $\mathcal{V}$, i.e., a pair of functors taking values in the category of bigraded abelian groups, one contravariant on $\mathcal{V}^*$,*

$$(Y \to X) \to \bigoplus_{p,q} H_Y^p(X, D(q)),$$

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