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ON WHITEHEAD'S ALGORITHM

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ABSTRACT. One can decide effectively when two finitely generated subgroups of a finitely generated free group F are equivalent under an automorphism of F. The subgroup of automorphisms of F mapping a given finitely generated subgroup S of F into a conjugate of S is finitely presented.

In two famous articles [**9**, **10**] which appeared in 1936, J. H. C. Whitehead, using the theory of three-dimensional handlebodies, proved that one can effectively decide when two n-tuples of cyclic words of a finitely generated free group F are equivalent by an automorphism of F. The proof of this result has been simplified successively [**7**, **3**] and the result itself has been immensely influential. Whitehead himself poses the problem of generalizing his theorem [**10**, p. 800]; namely he raises the question of deciding when two finitely generated subgroups of F are equivalent by an automorphism of F.

In 1974 McCool [**6**] deduced a profound consequence of Whitehead's theorem, proving that the stabilizer, in the automorphism group of F, of an n-tuple of cyclic words is finitely presented. Using graph-theoretic techniques we developed in [**1**] (the results of which were announced in [**2**]), we have succeeded both in settling Whitehead's question and in generalizing McCool's results.

Let A denote the automorphism group of F, and let $\mathcal{S}$ denote the set of conjugacy classes of finitely generated subgroups of F with its natural A action. Let $\mathcal{S}^n$ denote the cartesian product of n copies of $\mathcal{S}$ with diagonal A action.

THEOREM W. *There is an effective procedure for determining when two elements of $\mathcal{S}^n$ are in the same orbit of the A-action.*

THEOREM M. *The stabilizer in A of an element of $\mathcal{S}^n$ is finitely presented, and a finite presentation can be effectively determined.*

In this note we indicate briefly the ideas that go into the proofs of Theorems W and M. Full details will appear elsewhere.

We use the theory of *graphs* defined in [**2**]. A graph X is a nonempty set with involution, denoted $x \mapsto \bar{x}$, together with a retraction $\iota: X \to V(X)$ of X onto the fixed point set $V(X)$ of the involution. Morphisms of graphs preserve the involution and the retraction. The set $V(X)$ is called the set of *vertices* of

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