4. E. W. Cheney, *Introduction to approximation theory*, McGraw-Hill, New York, 1966.
5. P. J. Davis, *Interpolation and approximation*, Blaisdell, New York, 1963.
6. I. P. Natanson, *Constructive function theory*, vol. III, Ungar, New York, 1965.
7. G. Pólya, *Bemerkung zur Interpolation und zur Naherungstheorie der Balkenbiegung*, Z. Angew. Math. Mech. **11** (1931), 445–449.
8. I. J. Schoenberg, *On Hermite-Birkhoff interpolation*, J. Math. Anal. Appl. **16** (1966), 538–543.
9. J. F. Steffensen, *Interpolation*, 2nd ed., Chelsea, New York, 1950.
10. J. Suranyi and P. Turán, *Notes on interpolation*. I, Acta Math. Acad. Sci. Hungar. **6** (1955), 67–80.
11. J. L. Walsh, *Interpolation and approximation by rational functions in the complex domain*, 5th ed., Amer. Math. Soc. Colloq. Publ., no. 20, Amer. Math. Soc., Providence, R.I., 1969.
12. J. M. Whittaker, *Interpolatory function theory*, Stechert-Hafner, New York, 1964.

ELI PASSOW

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 3, November 1983

0273-0979/83 $1.00 + $.25 per page

The theory of Eisenstein systems, by M. Scott Osborne and Garth Warner, Academic Press, New York, 1981, xiii + 385 pp., $55.00. ISBN 0-1252-9250-3

The Laplace-Beltrami operator on the upper half-plane with respect to the hyperbolic metric is

$$\Delta = y^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right).$$

The arithmetic interest of the eigenfunctions of Δ invariant under the modular group $\Gamma = \mathrm{SL}(2, \mathbf{Z})$ and its congruence subgroups was signalled by Maass [**17**], who was inspired by earlier work of Hecke. If $\gamma \in \mathrm{GL}(2, \mathbf{Q})$ then $\Gamma_\gamma = \gamma^{-1}\Gamma\gamma \cap \Gamma$ is of finite index in Γ. Thus if $\det \gamma > 0$ so that γ also acts as a fractional linear transformation on the upper half-plane one can introduce the operator

$$T_\gamma : f \to \sum_{\delta \in \Gamma_\gamma \backslash \Gamma} f(\gamma \delta z), \qquad \mathrm{Im}\, z > 0.$$

It is called a Hecke operator. It commutes with Δ, and acts on its eigenspaces. The study of these operators and of those appearing in Hecke's work promises to be of considerable importance for diophantine problems, in particular for the investigation of the Dirichlet series to which the names of Artin and Hasse-Weil are attached. However the spectral theory of Δ on Γ-invariant functions is a purely analytic problem, of interest in its own right for any discrete subgroup Γ of $\mathrm{SL}(2, \mathbf{R})$ whose fundamental domain has finite volume. If the quotient of the upper half-plane by Γ is compact the spectrum is discrete, but otherwise there is a continuous spectrum and the corresponding eigenfunctions are called Eisenstein series.

If the quotient is not compact there are cusps. By way of illustration we may assume that ∞ is a cusp. This means that Γ contains a subgroup of the form

$$\Gamma_0 = \left\{ \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix} \middle| n \in \mathbf{Z} \right\}$$