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Topics in iteration theory, by György Targonski, Studia Mathematica, Skript 6, Vandenhoeck & Ruprecht, Göttingen and Zurich, 1981, 292 pp., (kart. DM 45, --), ISBN 3-5244-0126-9

Falstaff, to Prince Hal: “Oh, thou hast damnable iteration,
and art indeed able to corrupt a saint.”

Henry IV, Part 1: Act 1, Scene 2

As the quotation shows, iteration, in the general sense of repetition of an act, has been around a long time. Even as a mathematical discipline devoted to the study of the repeated composition of functions with themselves, iteration theory is rather old: it can be said to have begun with the activities of the Cambridge Analytical Society (Babbage, Herschel, Peacock) in the 1810s, and more particularly with the publication by Charles Babbage of his two-part *Essay towards the calculus of functions* in the Philosophical Transactions of the Royal Society in 1815 and 1816.

In that essay, as elsewhere, Babbage wrote ψ^n for the n th iterate of the function ψ , and posed the problem “Required the solution of

$$(1) \quad \psi^n x = x \dots”.$$

He observed that if ψ_1 is a solution of (1) and ψ_2 is defined by

$$(2) \quad \psi_2 = f^{-1} \circ \psi_1 \circ f,$$

where f is any invertible function whose range includes the domain and range of ψ_1 , then ψ_2 is also a solution of (1). Thus Babbage introduced the equivalence relation of *conjugacy of functions*. Conjugacy is a fundamental notion in iteration theory, for it is clear from (2) that all information about the iterative behavior of a function can be obtained from the corresponding behavior of any conjugate function. For example, for $0 < \lambda \leq 2$, let g_λ be the function defined on $[0, 1]$ by

$$g_\lambda(x) = 2\lambda x(1 - x).$$

Then g_λ is conjugate to the function h_λ defined on $[-\lambda, \lambda]$ by

$$h_\lambda(x) = x^2 - \lambda(\lambda - 1)$$