BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 3, November 1983

0273-0979/83 $1.00 + $.25 per page

The core model, by A. J. Dodd, London Mathematical Society Lecture Notes Series No. 61, Cambridge Univ. Press, New York, New York, 1982, xxxviii + 229 pp., $24.95. ISBN 0-5212-8530-5

Measurable cardinals have been around for a long time. Stanislaw Ulam [**U**] invented them in his salad days in Lwów, and they would emerge from time to time under various combinatorial guises. But they seemed at best a curiosity in a sideshow to the creative achievements of Kurt Gödel in mathematical logic. The driving life force in set theory at the time was the Continuum Problem, and Gödel [**Gö**] in 1938 established the consistency of Cantor's Continuum Hypothesis and also the Axiom of Choice by constructing the universe L of constructible sets. The reasons are not altogether clear for the prolonged lull that ensued, but at least during this period, the structural approach to *restrictions* of models of set theory initiated by Gödel's beautiful construction was systematized in the study of inner models (Sheperdson) and relative constructibility (Lévy). Then in 1963, the analyst Paul Cohen [**C**] established the independence of the Continuum Hypothesis and the Axiom of Choice. In his Forcing Method, the brilliant carpetbagger happened upon a remarkably fecund method for producing *extensions* of models of set theory. There was no lull this time, as fine mathematicians like Robert Solovay quickly perceived the possibilities abounding, and within a few years the general theory of Forcing was codified, and a cornucopia of relative consistency results were being fashioned.

This mainstream of new vitality in set theory was fed by another development, which is traced in more detail to establish the context for the book under review. In 1960, Dana Scott [**Sc**] proved that if there is a measurable cardinal, then the universe V of all sets is strictly larger than Gödel's constructible universe L, i.e. $V \neq L$. The usefulness of the ultraproduct construction in model theory was just beginning to be understood at the time when Scott struck on the idea of taking an ultrapower of the entire universe V. Not only did this simple but penetrating result firmly establish that new axioms can decide outstanding questions about the universe, but it quickly led to the intrinsic characterization of measurable cardinals as critical points (least ordinal moved) of elementary embeddings $j\colon V \to M$ of the universe V into some inner model M. If the work of Gödel and others had established a sacred tradition in logical syntax and formal structure, then Scott's result rescued