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A NONSTANDARD IDEAL OF A RADICAL BANACH ALGEBRA OF POWER SERIES

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1. Introduction. We will be concerned here with a result about certain radical Banach algebras of power series. Let $\mathbf{C}[[z]]$ denote the algebra of formal power series over the complex field $\mathbf{C}$. We say that a sequence of positive reals $\{w(n)\}$ is an *algebra* weight provided the following hold:

$$w(0) = 1 \quad \text{and} \quad 0 < w(n) \leq 1 \qquad (n \in \mathbf{Z}^+), \tag{1.1}$$

$$w(m+n) \leq w(m)w(n) \qquad (m, n \in \mathbf{Z}^+), \tag{1.2}$$

$$\lim_{n\to\infty} w(n)^{1/n} = 0. \tag{1.3}$$

If these conditions hold it is routine to check that

$$l^1(w(n)) \equiv \{y = \sum_{n=0}^{\infty} y(n)z^n : \sum_{n=0}^{\infty} |y(n)|w(n) < \infty\}$$

is both a subalgebra of $\mathbf{C}[[z]]$ and a radical Banach algebra with identity adjoined. The norm and multiplication are defined in the natural way (see [**3**, **4**, and **8**]). There are obvious closed ideals in $l^1(w(n))$:

$$M(n) \equiv \left\{ \sum_{k=0}^{\infty} y(k)z^k \in l^1(w(n)) : y(0) = y(1) = \cdots = y(n-1) = 0 \right\}$$

and, of course, the zero ideal. Such ideals are referred to as *standard* ideals. Any other closed ideals are denoted *nonstandard* ideals. It has been an open question for some time whether there exists any algebra weight $\{w(n)\}$ such that $l^1(w(n))$ contains a nonstandard ideal [**6**, p. 189], and the problem seems to go back to Šilov (a proposed solution appearing in the literature [**6**, p. 205] is in error). Interest has also been focused upon the quotient algebras $(l^1(w(n))/I)$, where I is assumed to be nonstandard, since these algebras are representative of all radical Banach algebras with power series generators [**1**]. Our result is the following:

THEOREM. *There exists an algebra weight $\{w(n)\}$ for which the algebra $l^1(w(n))$ contains a nonstandard ideal.*

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