BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 3, November 1983

INVARIANTS OF FORMAL GROUP LAW ACTIONS[1]

BY ROBERT M. FOSSUM

0. Introduction. In this note, k denotes a field of characteristic $p > 0$, and the letters T, X and Y are formal indeterminants. Let $F\colon k[[T]] \to k[[X,Y]]$ be a (fixed) one-dimensional formal group law [**Dieudonné, Hazewinkel, Lazard, Lubin**] of height $h \geq 0$. Let V denote a $k[[T]]$ module of finite length. Suppose $\operatorname{Ann}(V) = (T^n)$. Let $q = p^e$ denote the least power of p such that $n \leq q$. It follows that the symmetric powers $S_r(V)$ over k become $k[[T]]$-modules, annihilated by T^q, through the formal group law, viz: If $F(T) = X + Y + \sum_{i,j\geq 1} C_{ij}X^iY^j$ and f is in $S_t(V)$ and g is in $S_s(V)$, then

$$T(fg) = fTg + (Tf)g + \sum C_{ij}(T^if)(T^jg)$$

in $S_{t+s}(V)$.

Denote by $S.(V)$ the symmetric algebraic on V; so

$$S.(V) := \bigoplus_{r>0} S_r(V).$$

Then $S.(V)$ is a $k[[T]]$-module annihilated by T^q. The main purpose of this note is to announce and outline a proof of the theorem below. Several consequences and examples are included.

THEOREM. *Let $S.(V)^F := \{f \in S.(V) : Tf = 0\}$. The set $S.(V)^F$ is a normal noetherian subring of $S.(V)$ of the same Krull dimension. Furthermore, $S.(V)^F$ is factorial.*

1. An outline of the proof. To prove the Theorem one can consider two cases: $\operatorname{ht} F = h = 1$ and $\operatorname{ht} F = h \neq 1$. In case $\operatorname{ht} F = 1$, the action on $S.(V)$ is equivalent to an action of the cyclic group $\mathbf{Z}/qZ$ on $S.(V)$. This case is considered, in full generality, in [**Fossum, Griffith**] and [**Almkvist, Fossum**].

So consider the case $\operatorname{ht} F \neq 1$. It can be shown that there is a fixed power s of p, depending only on $\operatorname{ht} F$, such that $S.(V)^s \subset S.(V)^F$. Then one can extend the action of T to the field of fractions L of $S.(V)$ via

$$T(f/g) = T(fg^{s-1})/g^s.$$

Then one concludes that L^F is a field and

$$S.(V)^F = L^F \cap S.(V),$$

which shows that $S.(V)^F$ is a Krull domain and $S.(V)^F \supset k[S.(V)^s]$, which shows that $S.(V)^F$ is noetherian and $S.(V)$ is integral over $S.(V)^F$. Hence,

Received by the editors May 2, 1983.

1980 *Mathematics Subject Classification*. Primary 14L05, 14L30, 13H10, 13F15.

[1]This research has been supported by the National Science Foundation.