BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 3, November 1983

ON THE LOCAL LANGLANDS CONJECTURE IN PRIME DIMENSION

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Let F be a local field of residual characteristics p. Then it is a conjecture of Langlands [**JL**] that there should be a natural bijection between the set of n-dimensional semisimple representations of the absolute Weil-Deligne group of F and the set of irreducible admissible representations of $\mathrm{GL}_n(F)$. Some cases of this conjecture have been established [**He**, **JL**, **JPS**, **K**, **M**]. Here we announce further progress toward its verification.

To describe our results, we first note that by work of Bernstein and Zelevinsky [**Z**], one may restrict one's attention to irreducible representations of the Weil-Deligne group on the one hand and irreducible supercuspidal representations of $\mathrm{GL}_n(F)$ on the other hand. In this context, the conjecture says there should exist a bijection $\sigma \mapsto \pi(\sigma)$ of the set $\mathcal{A}_n^0(F)$ of equivalence classes of continuous, irreducible n-dimensional complex representations of W_F, the absolute Weil group of F, with the set $\mathcal{A}^0(\mathrm{GL}_n(F))$ of equivalence classes of admissible irreducible supercuspidal representations of $\mathrm{GL}_n(F)$. This bijection should satisfy the following conditions:

(1.01) $\epsilon(\pi(\sigma), \psi) = \epsilon(\sigma, \psi)$ (see [**D**, **GJ**] for definitions),

(1.02) $\pi(\sigma) \otimes \chi \circ \det = \pi(\sigma \otimes \chi)$ for all quasi-characters χ of F^x,

(1.03) $\omega_{\pi(\sigma)} = \det \sigma$, where $\omega_{\pi(\sigma)}$ is the central character of $\pi(\sigma)$.

We note that if $n = 1$, the existence of such a bijection is a restatement of the fundamental theorem of local classified theory [S]; thus when $n \geq 2$, the conjecture under consideration may be thought of as a nonabelian analogue of that theorem.

When $n \geq 2$ the construction of $\pi(\sigma)$ breaks naturally into two steps.

I. *Construction of $\pi(\sigma)$ when σ is induced from a representation of smaller dimension.* This construction is provided when $n = 2$ by decomposing the Weil representation of $\mathrm{SL}_2(F)$ (see [**JL**]). When $n = 3$, it is obtained by global methods [**JPS**]. When $p \nmid n$ then all n-dimensional irreducible representations σ of W_F are monomial, and one may use a representation which induces σ to construct a supercuspidal representation $\pi'(\sigma)$ of $\mathrm{GL}_n(F)$. This was first done by Howe [**Ho**], who conjectured that $\pi'(\sigma)$ satisfied (1.01)–(1.03). Recently, Moy [**M**] showed that a representation $\pi(\sigma)$ satisfying (1.01)–(1.03) may be obtained by a slight modification of Howe's construction and thus verified the Langlands conjecture in case $p \nmid n$ (one needs, however, that $\mathrm{char}\, F = 0$ in order that the map $\sigma \mapsto \pi(\sigma)$ be bijective).

When $p \mid n$, however, the above approach appears to fail.

Received by the editors June 27, 1983.

1980 *Mathematics Subject Classification*. Primary 12B25, 20G05; Secondary 22E50, 12B15.

[1]Supported in part by NSF Grant #MP575 07481.