subject of interpolation itself. The translation is lucid, professionally done, and reads well. All in all, the book is a welcome addition to the literature. Wordsworth, we are sure, would have approved.

COLIN BENNETT

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 2, September 1983

0273-0979/83 $1.00 + $.25 per page

Generalized solutions of Hamilton-Jacobi equations, by P. L. Lions, Research Notes in Mathematics, Vol. 69, Pitman Advanced Publishing Program, Boston, 1982, 317 pp., $24.95. ISBN 0-2730-8556-5

Introduction. The Hamilton-Jacobi equation is probably known to most engineers and physicists as a partial differential equation which pops up in the study of (Lagrangian or Hamiltonian) mechanics, yielding solutions of a system of ordinary differential equations, as its characteristics, after a variational procedure is used. It is also known, again through its relation to the calculus of variations, to people studying control theory, differential games, or other optimization problems, although it is sometimes referred to as the "Bellman equation" in these contexts.

The last thirty years has seen the rise of a new interest in the Hamilton-Jacobi equation. With the rise of computers and new numerical techniques, the failure of classical smooth solutions to describe physical situations except in limited (local) domains, and the needs of mathematical modeling, aerospace engineering, and other applications to have solutions described everywhere, many mathematicians have become interested in global solutions (whatever that means). As nearly the most general first order partial differential equation, and as an equation for which global results were possible, the Hamilton-Jacobi equation became a natural target for mathematicians studying global solutions.

In order to clarify the object of interest a little better, let us define the Hamilton-Jacobi equation. In its most familiar classical form, the Hamilton-Jacobi equation is

$$\partial u/\partial t + H(t, x, Du) = 0,$$

where H is a given function, called the Hamiltonian, x is in R^n, and Du denotes the gradient of the solution, u, with respect to x. Here t is a single variable (usually called "time"). The separation of the distinguished variable "t" from the gradient, Du, in H, makes the Hamilton-Jacobi equation much easier to handle than the general first order equation. The Cauchy (or initial value) problem is always noncharacteristic, thus amenable to solution. This same separation of t also makes the Hamilton-Jacobi equation essentially an evolution equation, thus allows a mass of evolution equation techniques to be brought to bear.

The Hamilton-Jacobi equation, as defined by Professor Lions, is

$$H(x, u, Du) = 0,$$