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MILNOR ALGEBRAS AND EQUIVALENCE RELATIONS AMONG HOLOMORPHIC FUNCTIONS[1]

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Let $\mathcal{O}_{n+1}$ denote the ring of germs at the origin of holomorphic functions $(\mathbf{C}^{n+1},0) \to \mathbf{C}$. As a ring $\mathcal{O}_{n+1}$ has a unique maximal ideal m, the set of germs of holomorphic functions which vanish at the origin. Let G_{n+1} be the set of germs at the origin of biholomorphisms $\phi\colon (\mathbf{C}^{n+1},0) \to (\mathbf{C}^{n+1},0)$. The following are three fundamental equivalent relations in $\mathcal{O}_{n+1}$.

DEFINITION 1. Let f, g be two germs of holomorphic functions $(\mathbf{C}^{n+1},0) \to (\mathbf{C},0)$.

(i) f is right equivalent to g if there exists a $\phi \in G_{n+1}$ such that $f = g \circ \phi$.

(ii) f is right-left equivalent to g if there exists a $\phi \in G_{n+1}$ and $\psi \in G_1$ such that $f = \psi \circ g \circ \phi$.

(iii) f is contact equivalent to g if $(V,0)$ is biholomorphic equivalent to $(W,0)$ where $V = \{z \in \mathbf{C}^{n+1}\colon f(z) = 0\}$ and $W = \{z \in \mathbf{C}^{n+1}\colon g(z) = 0\}$, i.e., there exists a $\phi \in G_{n+1}$ such that $\phi\colon (V,0) \to (W,0)$.

One of the natural and fundamental problems in complex analytic geometry is to tell when two germs of holomorphic functions $(\mathbf{C}^{n+1},0) \to (\mathbf{C},0)$ are equivalent in the sense of (i), (ii), or (iii) respectively in Definition 1. To answer the above problem, we need the following notations:

$$f^{-1}m_1 = \left\{\sum_{i\geq 1} a_i f^i\colon \sum_{i\geq 1} a_i t^i \text{ is a convergent power series in one variable}\right\},$$

$$\Delta(f) = \text{ideal in } \mathcal{O}_{n+1} \text{ generated by } \frac{\partial f}{\partial x_0}, \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n},$$

$$a(f) = \{g \in m\colon \Delta(g) \subseteq \Delta(f)\},$$

$$\underline{R}(f) = \{g \in m\colon g \text{ is right equivalent to } f\},$$

$$\underline{RL}(f) = \{g \in m\colon g \text{ is right-left equivalent to } f\},$$

$$\underline{K}(f) = \{g \in m\colon g \text{ is contact equivalent to } f\},$$

$$\underline{A}(f) = \{g \in m\colon \text{the moduli algebra of } g \text{ is isomorphic to the moduli algebra of } f, \text{ i.e., } \mathcal{O}_{n+1}/(f,\Delta(f)) \cong \mathcal{O}_{n+1}/(g,\Delta(g))\},$$

$$\underline{B}(f) = \{g \in m\colon \mathcal{O}_{n+1}/(f, m\Delta(f)) \cong \mathcal{O}_{n+1}/(g, m\Delta(g))\},$$

$$\underline{Q}(f) = \{g \in m\colon \text{The Milnor algebra of } g \text{ is isomorphic to Milnor algebra of } f, \text{ i.e., } \mathcal{O}_{n+1}/\Delta(f) \cong \mathcal{O}_{n+1}/\Delta(g)\}.$$

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