BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 2, September 1983

CHARACTERIZATION OF STRICTLY CONVEX DOMAINS BIHOLOMORPHIC TO A CIRCULAR DOMAIN

BY GIORGIO PATRIZIO[1]

Recently parabolic exhaustions have been used successfully to classify complex spaces (Stoll [**6** and **7**], Burns [**2**], P. Wong [**9**]). Here we define parabolic exhaustions for strictly convex domains in $\mathbf{C}^n$ and give necessary and sufficient conditions for such a domain to be biholomorphically equivalent to a circular domain or even to the ball.

The results presented in this note are part of the Ph.D. Thesis the author is completing under the direction of Professor W. Stoll. Details and more implications will appear at a later date.

A strictly convex domain $D \subset \mathbf{C}^n$ is a domain for which there exists a defining function whose real Hessian is strictly positive on $T_x(\partial D)$ for all $x \in \partial D$. Let Δ be the open unit disk in $\mathbf{C}$ and let S be the unit sphere in $\mathbf{C}^n$. Let $D \subset \mathbf{C}^n$ be a strictly convex domain and $p \in D$ be any point. For $b \in S$ Lempert [**4**] constructs an extremal map $F(\square, b)\colon \Delta \to D$ which is holomorphic with $F(0, b) = p$ and $F'(0, b) = \lambda$ where $1/\lambda > 0$ is the length of b in the infinitesimal Kobayashi metric of D at p. These conditions determine $F(\square, b)$ uniquely and the map extends smoothly to an embedding $F(\square, b)\colon \overline{\Delta} \to \overline{D}$. Also $F\colon \overline{\Delta} \times S \to \overline{D}$ is of class C^∞ and surjective. One and only one function $\tau\colon \overline{D} \to \mathbf{R}_+$ exists such that $\tau(F(z, b)) = |z|^2$ for $(z, b) \in \overline{\Delta} \times S$. Then τ is a continuous exhaustion of $\overline{D}$, positive and of class C^∞ on $\overline{D} \setminus \{p\}$. Also $\tau \equiv 1$ on ∂D. We refer to τ as the *Lempert exhaustion* of D at p.

THEOREM 1. *The Lempert exhaustion at p is strictly parabolic on $D_* = D \setminus \{p\}$, which means that on D_**
(1) $dd^c\tau > 0$,
(2) $dd^c \log \tau \geq 0$,
(3) $(dd^c \log \tau)^n \equiv 0$.

Properties (2) and (3) were proved by Lempert. For each r with $0 < r < 1$ the pseudoball $D(r) = \{x \in D \mid \tau(x) < r^2\}$ is a ball in the Kobayashi distance. By an argument of harmonic functions, it is shown that the Hessian of the defining function $\tau - r^2$ of $D(r)$ is positive on the tangent space of $\partial D(r)$ at every point of $\partial D(r)$. Hence $D(r)$ is strictly convex and $dd^c\tau > 0$ follows easily.

The strictly parabolic function τ defines a Monge-Ampère foliation on D which coincides with the foliation defined by F. More precisely if

$$X = X^\mu \frac{\partial}{\partial z^\mu} = \tau_{\overline{\nu}} \tau^{\overline{\nu}\mu} \frac{\partial}{\partial z^\mu}$$

Received by the editors November 18, 1982.
1980 *Mathematics Subject Classification*. Primary 32H99; Secondary 32F15.
[1]The author was supported by a grant from the Consiglio Nazionale delle Ricerche.