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FP_∞ GROUPS AND HNN EXTENSIONS[1]

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A group G is said to be of type FP_∞ if the $\mathbf{Z}G$-module $\mathbf{Z}$ admits a projective resolution (P_i) of finite type (i.e., with each P_i finitely generated). If G is finitely presented, this is equivalent by Wall [**5**, **6**] to the existence of an Eilenberg-Mac Lane complex $K(G,1)$ of finite type (i.e., with finitely many cells in every dimension). Up to now, all known torsion-free groups of type FP_∞ have had finite cohomological dimension; in fact, they have admitted a *finite* $K(G,1)$-complex. We announce here the first known example of a torsion-free group of type FP_∞ with infinite cohomological dimension. This solves Wall's problem F11 [**7**]. We show in addition that our group, which we denote by F, satisfies $H^n(F,\mathbf{Z}F)=0$ for all n. As far as we know, F is also the first example of an FP_∞ group with this property. The vanishing of $H^*(F,\mathbf{Z}F)$ is a consequence of results of independent interest concerning the cohomology of HNN extensions (or, more generally, fundamental groups of graphs of groups) with free coefficients.

The group F is defined by the presentation $\langle x_0,x_1,x_2,\ldots; x_i^{-1}x_nx_i=x_{n+1}$ for $i<n\rangle$. It has previously arisen in two contexts: (i) finitely presented infinite simple groups (R. J. Thompson [unpublished]); and (ii) unsplittable free-homotopy idempotents (Freyd and Heller [**3**], Dydak and Minc [**2**]). F was previously known to be finitely presented, torsion-free, and of infinite cohomological dimension. (In fact, F has a subgroup which is free abelian of infinite rank.) Our contribution, therefore, is

THEOREM 1. *The group F described above is of type FP_∞ and satisfies* $H^*(F,\mathbf{Z}F)=0$.

The proof that F is of type FP_∞ goes as follows. Let $\phi\colon F\to F$ be the shift map, $\phi(x_i)=x_{i+1}$. Note that $\phi^2=T_{x_0}\circ\phi$, where T_{x_0} is the conjugation map $x\mapsto x_0^{-1}xx_0$; thus ϕ is idempotent up to conjugacy. We construct the universal example of a semicubical complex K with (a) a free right F-action; (b) a basepoint-preserving cubical endomorphism $\psi\colon K\to K$ compatible with ϕ; and (c) a homotopy from ψ^2 to $\rho_{x_0}\circ\psi$ compatible with ϕ^2, where $\rho_{x_0}(e)=ex_0$. (The motivation for this comes from (ii) above; K should be thought of as the universal cover of a complex with a free-homotopy idempotent, and ψ should be thought of as a lift of the idempotent to K.) We prove by a direct combinatorial argument that K is acyclic; the chain complex C of K is therefore a free resolution of $\mathbf{Z}$ over $\mathbf{Z}F$. Unfortunately, C is not of finite type. But we are able to find a contractible chain subcomplex $D\subset C$ such

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