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ENTROPIES AND FACTORIZATIONS OF TOPOLOGICAL MARKOV SHIFTS

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1. Markov shift entropies. Let A be a nonnegative integral matrix. A well-known construction [**7**] associates to A a homeomorphism σ_A of a totally disconnected compact space called a topological Markov shift, or subshift of finite type. Such Markov shifts play a central role in topological dynamics (see [**3**]), the investigation of Smale's Axiom A diffeomorphisms [**6**], and coding theory [**1**]. We announce here a characterization of the possible values for the topological entropy of such Markov shifts, answering a question raised in [**2**]. Furthermore, these values possess an arithmetic structure which, together with the isomorphism theorem of Adler and Marcus [**2**], yields an analogue of prime factorization for Markov shifts up to almost topological conjugacy. Details and applications of these results will appear elsewhere.

We shall always assume A to be aperiodic, i.e. some power of A is strictly positive. The topological entropy of σ_A is $\log\lambda$, where λ is the spectral radius of A [**5**]. Perron-Frobenius theory [**4**] shows that λ must be an algebraic integer > 1 whose other conjugates have absolute value $< \lambda$. Call an algebraic integer with these properties a Perron number. Our principal result shows these are the only restrictions on Markov shift entropies.

THEOREM 1. *If λ is a Perron number, then there is a nonnegative aperiodic integral matrix whose spectral radius is λ.*

SKETCH OF PROOF. If λ is Perron, let B be the $d \times d$ companion matrix of the minimal polynomial over $\mathbf{Q}$ of λ. The main difficulty occurs when B has no invariant d-sided cones, e.g. when $\operatorname{tr} B < 0$. This is overcome by finding invariant surfaces for B curved towards the dominant eigendirection.

The real Jordan form for B decomposes $\mathbf{R}^d$ into direct sum of the 1-dimensional dominant eigenspace $D = \mathbf{R}w$ for λ, a collection $\mathcal{E} = \{E\}$ of 1- or 2-dimensional eigenspaces with $||Bx|| = \gamma_E||x||(x \in E)$ for constants $\gamma_E > 1$, and another collection $\mathcal{F} = \{F\}$ of eigenspaces with $||Bx|| = \gamma_F||x||(x \in F)$, $\gamma_F \leq 1$. If $G = D$, E, or F, let π_G be the B-equivariant projection from $\mathbf{R}^d$ to G. We will use $\pi_D\colon \mathbf{R}^d \to \mathbf{R} \cong D$ normalized by $\pi_D w = 1$. Put $\pi_C = I - \pi_D$.

Fix $\theta > 0$, and put

$$K_\theta = \{x \in \mathbf{R}^d : \pi_D x > \theta||\pi_C x||\}, \qquad K_\theta(r) = \{x \in K_\theta : \pi_D x \leq r\}.$$

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