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STEENROD-SITNIKOV HOMOLOGY FOR ARBITRARY SPACES

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1. Introduction. In order to establish an Alexander duality theorem for compact subsets of S^n, N. E. Steenrod introduced in 1940 a new type of homology of metric compacta. The same problem led K. A. Sitnikov in 1951 to an equivalent theory. In 1960 J. Milnor [**7**] gave an axiomatic characterization of the Steenrod-Sitnikov homology. Several authors extended the theory to the case of Hausdorff compact spaces (see, e.g., [**8**, **9**, **7** and **1**]).

The purpose of this announcement is to define a Steenrod-Sitnikov homology theory for arbitrary topological spaces. We refer to it as strong homology. It is obtained by first developing a strong homology of inverse systems. The transition from spaces to systems is achieved by means of ANR-resolutions, a new tool developed by S. Mardešić in [**5**] (also see [**6**]). Strong homology groups of a space are then defined as strong homology groups of any one of its ANR-resolutions. It is a consequence of our approach that strong homology is actually a functor on the strong shape category SSh introduced in [**4**].

2. Strong homology of inverse systems. We consider only inverse systems of topological spaces and maps $\mathbf{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ over directed cofinite sets. By a map of systems $f\colon \mathbf{X} \to \mathbf{Y} = (Y_\mu, q_{\mu\mu'}, M)$ we mean an increasing function $\varphi\colon M \to \Lambda$ and a collection of maps $f_\mu\colon X_{\varphi(\mu)} \to Y_\mu$, $\mu \in M$, satisfying

$$f_\mu p_{\varphi(\mu)\varphi(\mu')} = q_{\mu\mu'} f_{\mu'}, \qquad \mu \le \mu'. \tag{1}$$

For a fixed Abelian group G we associate with $\mathbf{X}$ a chain complex $C_\#(\mathbf{X}; G)$, defined as follows. Let Λ^n, $n \ge 0$, denote the set of all increasing sequences $\lambda = (\lambda_0, \ldots, \lambda_n)$ from Λ. A strong p-chain of $\mathbf{X}$, $p \ge 0$, is a function x, which assigns to every $\lambda \in \Lambda^n$ a singular $(p+n)$-chain $x_\lambda \in C_{p+n}(X_{\lambda_0}; G)$. The boundary operator $d\colon C_{p+1}(\mathbf{X}; G) \to C_p(\mathbf{X}; G)$ is defined by the formula

$$(-1)^n (dx)_\lambda = \partial(x_\lambda) - p_{\lambda_0\lambda_1\#} x_{\lambda_0} - \sum_{j=1}^{n} (-1)^j x_{\lambda_j}; \tag{2}$$

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