BOOK REVIEWS

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Gauge theory and variational principles, by David D. Bleecker, Global Analysis: Pure and Applied, Series A, Addison-Wesley Publishing Company, Inc., Reading, Mass., 1981, xviii + 180 pp., $17.50, ISBN 0-2011-0096-7

Over the past fifteen years a consensus has developed among elementary particle physicists that most, if not all, interactions between the fundamental particles of nature are described by gauge theories. During the last decade it has become clear that a gauge theory, at least in its classical aspects, is in fact a theory of connections and curvatures in principal bundles and associated vector bundles over (suitably compactified) spacetime manifolds, and this recognition has led to some nontrivial mathematical results. The book by Bleecker is a timely introduction to the differential geometry and variational principles of classical gauge theories.

Before proceeding to a brief review of the book, I would like to make a general remark on the strange interactions between mathematics and physics, and what Wigner called the "unreasonable effectiveness of mathematics" in modern physics, [**Wigner, 1960**]. One must begin to wonder why, from time to time, the disciplines of theoretical physics and modern mathematics drift apart, seem to develop in relative independence from each other, and then suddenly find domains of overlap and cross-fertilization. This happened before in this century (I have in mind the parallel developments of quantum mechanics, Hilbert space theory, and the theory of group representations, the developments of operator algebra theory and algebraic quantum field theory and statistical mechanics, inverse scattering methods and solitons, and most intriguing of all, the pervasiveness of modern differential geometry and topology in developments in classical mechanics, general relativity, and now gauge theories; another recent related development is "supersymmetry" also known as the theory of graded Lie algebras, which is rapidly making contacts with gauge theories both in its physical, and in its mathematical aspects). One can only wonder about the deeper reasons of these developments and look forward to further cross-fertilization.

The term *gauge invariance* (*Eichinvarianz* in the original) together with the fundamental idea of a gauge theory was introduced by Hermann [**Weyl, 1918**] in an attempt to incorporate electromagnetism into a unified theory of gravitation and electromagnetism. His idea of "gauging" consisted in extending Einstein's principle of relativity by assuming that the scale of length can vary smoothly from point to point in spacetime. This led to the introduction of a one-form $A = A_\mu dx^\mu$ (identified with the electromagnetic four-potential) and