BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 9, Number 1, July 1983

THE SULLIVAN CONJECTURE

BY HAYNES MILLER[1]

THEOREM 0. *Let G be a locally finite group with classifying space BG and let X be a connected finite dimensional CW complex. Then the space of pointed maps from BG to X is weakly contractible:* $\pi_i(\mathrm{map}_*(BG,X))=0$ *for all $i\geq 0$. In particular, every map from BG to X is null-homotopic.*

A group is locally finite if it is a direct limit of finite groups. When G is of order 2, this theorem represents an affirmative resolution of part of a conjecture of Dennis Sullivan [**11**]. In this announcement I will sketch a proof of this theorem; details will appear in due course.

Theorem 0 is a straightforward consequence of the following three results.

THEOREM 1. *If X is a connected CW complex of finite category and G is a torsion group, then every map $f\colon BG\to X$ is trivial in π_1.*

THEOREM 2. *If X is a nilpotent space such that $H_*(X;\mathbf{F}_p)$ is bounded—that is, $H_i(X;\mathbf{F}_p)=0$ for all large i—then* $\mathrm{map}_*(BZ_p,X)$ *is weakly contractible.*

THEOREM 3. *Let X be a nilpotent space and G a locally finite group. If* $\mathrm{map}_*(BZ_p,X)$ *is weakly contractible for every prime p occurring as the order of an element of G, then* $\mathrm{map}_*(BG,X)$ *is weakly contractible.*

Theorem 1 is an exercise in K-theory and covering spaces. For the remaining theorems, it is convenient to work simplicially; the topological results follow from standard comparison theorems. So "space" will always mean "simplicial set", usually assumed fibrant.

PROOF OF THEOREM 2. The crux is the following theorem, which distills results of Bousfield and Kan [**5**], and Dror, Dwyer, and Kan [**7**].

THEOREM 2.1. *Let W be a connected space such that $\overline{H}_*(W;\mathbf{Z}[\frac{1}{p}])=0$, and let X be nilpotent. Then the natural map $X\to\mathbf{F}_{p\infty}X$ to the Bousfield-Kan $\mathbf{F}_p$-completion induces a weak equivalence*

$$\mathrm{map}_*(W,X)\to\mathrm{map}_*(W,\mathbf{F}_{p\infty}X).$$

Received by the editors February 2, 1983.

1980 *Mathematics Subject Classification*. Primary 55S35, 55T10; Secondary 18G55, 13D03, 20J10.

[1]Partially supported by the Alfred P. Sloan Foundation and NSF grants MCS-8002780 and MCS-8108814(A01).