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SOME RESULTS IN HARMONIC ANALYSIS IN $\mathbf{R}^n$, FOR $n \to \infty$

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1. Introduction. The purpose of this note is to bring to light some further results whose thrust is that certain fundamental estimates in harmonic analysis in $\mathbf{R}^n$ have formulations with bounds independent of n, as $n \to \infty$.

It was shown previously (see [**3**, and **4**]) that for the basic maximal function M, defined by

$$Mf(x) = \sup_B \frac{1}{m(B)} \int_B |f(x-y)|\, dy$$

(with the sup taken over all balls B centered at the origin), one has

$$\|M(f)\|_p \leq A_p \|f\|_p, \qquad 1 < p \leq \infty, \tag{1}$$

with bound A_p independent of n. Here we show that the analogue holds for the basic singular integrals, the "Riesz transforms" and their powers. While such results may have some interest on their own (the usual proofs give bounds which increase rapidly as $n \to \infty$), their validity raises certain further general questions and leads to speculation which we shall indulge in briefly at the end of this note.

2. The theorem. In $\mathbf{R}^n$ we define the familiar Riesz transforms by $(R_j f)\hat{}\,(\xi) = i(\xi_j/|\xi|)\hat{f}(\xi)$, $j = 1, \ldots, n$, and write $R = (R_1, \ldots, R_n)$; also $|R(f)(x)|$ will stand for $(\sum_{j=1}^n |R_j(f)(x)|^2)^{1/2}$.

THEOREM.

$$\|R(f)\|_p \leq A_p \|f\|_p, \qquad 1 < p < \infty,$$

with A_p independent of n.

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