

FIXED POINT THEORY AND NONLINEAR PROBLEMS

BY FELIX E. BROWDER

Introduction. Among the most original and far-reaching of the contributions made by Henri Poincaré to mathematics was his introduction of the use of topological or “qualitative” methods in the study of nonlinear problems in analysis. His starting point was the study of the differential equations of celestial mechanics, and in particular of their periodic solutions. His work on this topic began with his thesis in 1879, and was developed in detail in his great three-volume work, *Méthodes nouvelles de la mécanique céleste*, which appeared in the early 1890s and summarized his many memoirs of the intervening period. It continued until his memoir shortly before his death in 1912 in which he put forward the unproved fixed point result usually referred to as “Poincaré’s last geometric theorem”.

The ideas introduced by Poincaré include the use of fixed point theorems, the continuation method, and the general concept of global analysis. The writer’s acquaintance with Poincaré’s influence came through contact with Solomon Lefschetz and Marston Morse, both of whom were very explicit as to the role of Poincaré as an initiator in this direction of mathematical development. In 1934, in the Foreword to his Colloquium volume on *The calculus of variations in the large*, Morse had put this forward very forcefully in the first paragraph:

“For several years the research of the writer has been oriented by a conception of what might be termed macro-analysis. It seems probable to the author that many of the objectively important problems in mathematical physics, geometry, and analysis cannot be solved without radical additions to the methods of what is now strictly regarded as pure analysis.

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