Osterwalder-Schrader positivity property $E_{\varepsilon_\Lambda}\theta X \cdot X \geqslant 0$ where θ is the reflection in a hyperplane Π.

Once again, quantum physics has turned into probability theory. The theory of random functions indexed by higher-dimensional spaces has been largely the province of those doing statistical mechanics. The imaginary time approach to constructive quantum field theory led to an extraordinarily fertile interaction of quantum physics and statistical mechanics.

The ultraviolet problem in two dimensions is relatively easy. Glimm and Jaffe do not give details in this book of their solution of the much harder ultraviolet problem in three dimensions. A variety of techniques from statistical mechanics is used to control the infrared limit, most notably correlation inequalities and cluster expansions. In Part I, among other things, the authors give beautiful expositions of these techniques in the simplest cases, and this eases the way for the quite difficult applications to field theory.

Once the Euclidean random field ϕ has been constructed, the corresponding quantum field may be obtained. There is one proviso: ϕ may not be ergodic under translations, which means that the quantum field may not have a unique vacuum. This is not a technicality. Indeed, ergodicity may fall, leading to a phase transition. The successes of constructive quantum field theory discussed here by Glimm and Jaffe have gone far beyond showing the existence of models—phase transitions, broken symmetry, particle structure, the scattering matrix, and other topics of physical interest have been thoroughly explored.

Functional integration has been far more successful in quantum physics than those of us who first learned the purely Hilbert-space approach ever dreamed. There is a mystery in this. Perhaps the mathematical trick of analytical continuation in time, which is applicable in some but not all situations, is not the key to the mystery. Perhaps probability theory has been so successful because the phenomena of quantum physics are inherently random phenomena. Whether this speculation is correct, only non-imaginary time will tell.

EDWARD NELSON

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Real elliptic curves, by Norman L. Alling, Mathematics Studies, Vol. 54, North-Holland Publishing Company, Amsterdam, 1981, xii + 350 pp., $36.25 US/Dfl. 85.00 paperback. ISBN 0-4448-6233-1

The author has (in collaboration with N. Greenleaf [**2**]) developed an interesting approach to real elliptic curves as an object of study in their own right, and not as a special case of complex analysis (as the universal imbedding subject). The theory was present in classical literature going back to 1882 (Klein [**8**]), and the historical context has stimulated the author to make a scholarly survey of elliptic functions from even before Gauss. This survey