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The representation theory of the symmetric group, by Gordon James and Adalbert Kerber, Encyclopedia of Mathematics and its Applications, vol. 16, Addison-Wesley, Reading, Mass., 1981, xxviii + 510 pp., $44.50.

There are many excellent texts on the representation-theory of finite groups, e.g. [**21**] for the 'ordinary' theory (over fields of characteristic 0) and [**6**, **8**, **14**, **39**], for the modular theory. The representation-theory of the symmetric groups S_n cannot, at present, be considered merely a specialization of the more general theory; there is a rich accumulation of concepts and theorems in the theory of S_n, whose analogs for arbitrary finite groups do not exist (or have not yet been found). The results of this special theory, not only have an intrinsic interest and beauty, but have applications in chemistry, physics and areas of mathematics as diverse as algebraic geometry (via flag manifolds, determinantal varieties and the Schubert calculus), classical invariant theory, rings with polynomial identity, multivariate statistics (through the work of A. T. James—*not* an author of the text under review!—and many followers; cf. [**13**, Chapters 12 and 13]) and, of course, combinatorics (in particular, via the Robinson-Schensted correspondence and the Redfield-Pólya enumeration-theory).

Two quite different approaches have given rise to most of this theory of S_n:

On the one hand, Frobenius determined the character-table of S_n utilizing certain symmetric polynomials (then called 'bi-alternants', now usually called 'Schur functions' or 'S-functions') and their known properties (which had been intensively investigated by Jacobi and others in the mid-nineteenth century). His student, Schur, extended this work, using it in his thesis to study the representation-theory of $GL(n)$. This approach studies the representations of S_n in terms of their characters, and in the context of the theory of symmetric polynomials.

On the other hand, Alfred Young, in his work on classical invariant theory, was led to study what he called 'Quantitative Substitutional Analysis' (the common title of the brilliant series of nine papers where he developed this approach to invariant- and representation-theory; cf. [**51**]); this theory involves many fundamental constructions, of which the best-known are the Young standard tableaux and the Young idempotents. Young's presentation is very condensed, and a small portion of his work has been popularized in Rutherford [**43**]; the bulk of Young's profound work has not yet been fully understood, in the reviewer's opinion. As developed also by Specht [**45**] and Garnir [**16**], this second approach deals with representations of S_n by constructing explicit S_n-modules, rather than dealing primarily with the associated characters.

(To do full justice to the role played by $GL(n)$ in the theory of S_n, which cannot be done in this review, would require discussion of a third important