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The structure of locally compact Abelian groups, by David L. Armacost, Pure and Applied Mathematics: A Series of Monographs and Textbooks, vol. 68, Dekker, New York, 1981, vii + 168 pp., $23.75.

Fourier analysis is undoubtedly one of the most important branches of mathematical analysis. It began as a separate area of study in the 19th century with the investigation of convergence of Fourier series; it has developed widely in the past 150 years; at least since the publication of Weil's book [**7**], mathematicians have realized that some of the main results in the subject extend naturally to locally compact Abelian (LCA) groups. It is reasonable, then, for harmonic analysts to ask what an LCA group can look like.

Suppose that we want to classify LCA groups. What likely paths are open?

1. The fundamental theorem on the structure of LCA groups says that any such group G can be written as $\mathbf{R}^n \times G_0$, where G_0 has a compact open subgroup. Moreover, both factors are unique up to isomorphism. Since we presumably understand $\mathbf{R}^n$, this means that we need only classify those LCA groups with compact open subgroups—discrete groups, for instance. A glance at any book on infinite Abelian groups ([**3** or **4**], for example) will show that classifying such groups is a hopeless task.

2. It might still be possible to classify the compact Abelian groups, thus making a dent in the general classification. Unfortunately, the Pontryagin Duality Theorem gives a bijective correspondence between compact and discrete Abelian groups. Thus this classification problem is also intractable.

3. The structure theorem says that (modulo factors of $\mathbf{R}^n$) every LCA group G_0 is the extension of a compact group K by a discrete group D. We could simply hope to classify such extensions, thus reducing the general classification problem to the two special problems mentioned above plus one problem in group cohomology. Indeed, we might use the Pontryagin Duality Theorem to describe the extension in terms of the discrete groups D and $K^\wedge$ ($K^\wedge$ is the dual of K). Thus the extension problem becomes a purely algebraic one (no topology involved). The formal computations are not hard; they are given in [**2**] (where they are used for a somewhat specialized purpose). However, this approach introduces two related problems. First of all, the compact open subgroup K of G_0 is generally not uniquely determined; thus the above approach gives a large number of different descriptions of the same group. Secondly, inequivalent extensions of K by D can give rise to isomorphic copies of G_0. (For example, there are two isomorphism classes of groups of order p^2, but $H^2(\mathbf{Z}/p\mathbf{Z}, \mathbf{Z}/p\mathbf{Z}) \cong \mathbf{Z}/p\mathbf{Z}$.) For the moment, then, this approach seems unsatisfactory.

4. Since the general classification problem seems hopeless, we might examine LCA groups with special properties. For instance, we might look at connected LCA groups. By the structure theorem, we need only consider compact