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A UNITED-SET FORMULA

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Let a morphism $f: X \to Y$ of algebraic varieties be given. A *united set* or *united k-tuple*[2] for f is a k-tuple $x_1, \ldots, x_k$ of distinct points on (or "infinitely near") X, such that $f(x_1) = \cdots = f(x_k)$. The purpose of this note is to announce an enumerative formula, valid under a restrictive hypothesis, for the united k-tuples of a map, i.e., a formula for the rational equivalence (or homology) class of a suitable cycle which parameterizes them. This yields as special cases formulas for the united k-tuples which contain a k_1-tuple, a k_2-tuple, etc. of mutually infinitely-near points. For our united-k-tuple cycle even to be defined, the morphism f has to admit a certain kind of "resolution" (essentially it must factor through a "generic" map into a variety fibred by smooth curves over Y). Our result is sufficient, however, to yield formulas for the lines having prescribed contacts with a given projective variety having "generic" singularities and arbitrary dimension and codimension; these in turn yield formulas for the Thom-Boardman-Roberts singularity schemes [**8**] of a generic projection of such a variety. Classically such formulas were known for curves, for surfaces in $\mathbf{P}^3$, and in a few other cases, cf. [**1**]. Some recent results were obtained by Lascoux [**6**], Roberts [**9**] and LeBarz [**7**]. Our result yields new formulas even for surfaces in $\mathbf{P}^4$. For a modern account of these and related matters, see Kleiman's surveys [**3**, **5**].

Admittedly, the hypothesis of existence of a "resolution" is a severe restriction on the morphism f. I am hopeful, however, that by pursuing further the same principles as in this paper, I will eventually obtain a united-set formula valid without such a restriction, and which would moreover be completely "intrinsic", in the sense of taking place on a suitable space associated solely to X (which is not the case with the present formula).

We shall work in the category of complete (usually nonsingular) varieties over a field. Everything goes through with no change, however, in the category of compact complex manifolds.

1. Set-up. Fix a morphism $f: X \to Y$ of nonsingular varieties, and put $m = \dim X$, $n = \dim Y$. A *resolution* of f is a diagram

$$\begin{array}{ccc} & \tilde{f} & Z \\ X & \nearrow \!\!\!\! \searrow & \downarrow \pi \\ & f & Y \end{array}$$

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[2]This term is *not* consistent with the classical one of united point of a correspondence.