RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 8, Number 2, March 1983

INSTANTONS, DOUBLE WELLS AND LARGE DEVIATIONS

BARRY SIMON[1]

ABSTRACT. We find the leading asymptotics of the exponentially small splitting of the two lowest eigenvalues of $-\frac{1}{2}\Delta + \lambda^2 V$ in the limit as $\lambda \to \infty$ where V is a nonnegative potential with two zeros.

In this note, we consider eigenvalues of a Schrödinger operator $-\frac{1}{2}\Delta + \lambda^2 V$ in the limit $\lambda \to \infty$ (which is quasiclassical since up to a factor of $\hbar^2$, $-\hbar^2\Delta + V$ has this form with $\hbar = \lambda^{-1}$ going to zero). The method can deal with eigenvalues other than the lowest, with multiple minima (even manifolds of minima) or degenerate minima. In this note, for simplicity we discuss only the two lowest eigenvalues, $E_0(\lambda)$ and $E_1(\lambda)$, with corresponding normalized eigenvectors $\Omega_0(\lambda)$, $\Omega_1(\lambda)$. More general situations and detailed proofs will appear elsewhere **[13]**. We will suppose the following about V. (i) V is C^∞, (ii) $V(x) \geq 0$ for all x and $\underline{\lim}_{|x|\to\infty} V(x) > 0$, (iii) V vanishes at exactly two points a and b and at these points $\partial^2 V/\partial x_i \partial x_j$ is strictly positive definite.

Under these circumstances, one can prove (see e.g. **[12]**) that ψ_0 is concentrated as $\lambda \to \infty$ near the points a, b and that $E_i(\lambda)/\lambda$ has a finite nonzero limit. We want also to suppose that for all ϵ small and for $y = a\, and$ for $y = b$

$$\underline{\lim}_{\lambda\to\infty} \int_{|x-y|\leq\epsilon} |\psi_0(\lambda, x)|^2 dx > 0. \tag{1}$$

One case where (1) holds is when there is a symmetry of order 2 (such as reflection) which leaves V and $-\Delta$ invariant, so that the limit is $\frac{1}{2}$ for $y = a$ or b.

Under these circumstances, one expects that the splitting of E_1 and E_0 will be governed by tunneling and one goal here is to obtain multidimensional

Received by the editors October 21, 1982 and, in revised form, November 29, 1982.
1980 *Mathematics Subject Classification*. Primary 35P15, 81H99; Secondary 60J65.
[1]Research partially supported by USNSF under grant MCS-81-20833.