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ARITHMETIC GROUPS ACTING ON COMPACT MANIFOLDS

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In this note we announce results concerning the volume preserving actions of arithmetic subgroups of higher rank semisimple groups on compact manifolds. Our results can be considered as the first rigidity results for homomorphisms of these groups into diffeomorphism groups and show a sharp contrast between the behavior of actions of these groups and actions of free groups.

Let G be a connected semisimple Lie group with finite center, such that every simple factor of G has R-rank ≥ 2. Let $\Gamma \subset G$ be a lattice. Then Γ is known to be finitely generated and arithmetic (the latter being a result of Margulis). If Γ is cocompact, then by standard arithmetic constructions one may have homomorphisms $\Gamma \to K$ where K is a compact Lie group and the image of Γ is dense. Thus Γ acts isometrically (and ergodically) on the homogeneous spaces of K. Our first result concerns perturbations of isometric actions. We recall that if one has an isometric diffeomorphism of a Riemannian manifold, then a perturbation of this diffeomorphism is not likely to be isometric. In fact hyperbolicity rather than isometry is typical of properties of a diffeomorphism that are preserved under a perturbation. The same remarks obviously apply as well to actions of free groups. However, with Γ as above we have the following.

THEOREM A. *Let M be a compact Riemannian manifold,* $\dim M = n$. *Set* $r = n^2 + n + 1$. *Assume Γ acts by smooth isometries of M. Let $\Gamma_0 \subset \Gamma$ be a finite generating set. Then any volume preserving action of Γ on M which*

(i) *for elements of Γ_0 is a sufficiently small C^r-perturbation of the original action, and*

(ii) *is ergodic,*

actually leaves a C^0-Riemannian metric invariant. In particular, there is a Γ-invariant topological distance function and the action is topologically conjugate to an action of Γ on a homogeneous space of a compact Lie group K defined via a dense range homomorphism of Γ into K.

We conjecture that without the ergodicity assumption one can still deduce the existence of a Γ-invariant C^0-Riemannian metric. The next result, proved by similar methods (together with results of Margulis [2] and Raghunathan [3]), is in the direction of the following conjecture.

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