BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 8, Number 1, January 1983

ON THE SCHROEDINGER CONNECTION

BY R. E. MEYER AND J. F. PAINTER[1]

A new and more direct approach to the connection of wave amplitudes across turning points and singular points of wave- and oscillator-equations has been found. It emphasizes and extends the view [1] that the connection formulae are an asymptotic expression of the branch structure of the singular point and reveals an unexpected two-variable structure even close to such points. It also extends turning-point theory to new classes of *irregular* points of differential equations

$$\epsilon^2 d^2 w/dz^2 + p(z)w(z) = 0 \tag{1}$$

with constant ϵ and analytic $p(z)$ that are physical Schroedinger equations in the sense that the concept of wavelength (or period) can be defined.

A natural (Liouville-Green) variable x measured in units of local wavelength is then also definable. Limit points of singular points of $p(z)$ will be excluded, as will singular points artificially introduced to represent radiation conditions. Any turning- or singular point of $p(z)$ must then correspond to a definite x, and if those points be identified with $z = 0$ and $x = 0$, respectively, then

$$x = \frac{i}{\epsilon}\int_0^z [p(t)]^{1/2}dt \tag{2}$$

must exist, at least as a multivalued function, on a neighborhood of zero.

For a clear theory, this hypothesis should be rephrased in terms of the natural variable: an analytic branch $r(x)$ of $p^{1/4}$ is defined on a Riemann surface element D about $x = 0$ which includes $-\pi < \arg x < 2\pi$ (i.e., three Stokes sectors, in turning-point terminology) so that $idz/dx = \epsilon/r^2$ is integrable at $x = 0$.

In the natural variable, with $w(z) = y(x)$, (1) takes the form

$$y'' + 2r^{-1}r'y' = y, \qquad r'/r = (\epsilon/2ip)d(p^{1/2})/dz, \tag{3}$$

and wave modulation is therefore controlled by r'/r; since $p = p(z)$, also ϵx depends only on z, by (2), and xr'/r depends on x and ϵ only through the product ϵx, by (3). Turning points and singular points of (1) are all singular points of (3), and when they do not lie on the real axes of z or x, physics places no further, general restriction on their nastiness. For the results here reported, the following, secondary hypothesis has been found sufficient: a limit

Received by the editors September 11, 1981.
1980 *Mathematics Subject Classification*. Primary 34E20; Secondary 41A60.
[1]Sponsored by the United States Army under Contract NO. DAAG29-80-C-0041. The work was supported partially by the National Science Foundation under Grant No. MCS 8001960 and by the Wisconsin Alumni Research Foundation.