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DYER-LASHOF OPERATIONS IN K-THEORY

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Dyer-Lashof operations were first introduced by Araki and Kudo in [**1**] in order to calculate $H_*(\Omega^n S^{n+k}; Z_2)$. These operations were later used by Dyer and Lashof to determine $H_*(QY; Z_p)$ as a functor of $H_*(Y; Z_p)$ [**5**], where $QY = \bigcup_n \Omega^n \Sigma^n Y$. This has had many important applications. Hodgkin and Snaith independently defined a single secondary operation in K-homology (for p odd and $p = 2$ respectively) which was analogous to the sequence of Dyer-Lashof operations in ordinary homology [**7, 13**], and this operation has been used to calculate $K_*(QY; Z_p)$ when Y is a sphere or when $p = 2$ and Y is a real projective space [**11, 12**]. In this note we describe new primary Dyer-Lashof operations in K-theory which completely determine $K_*(QY; Z_p)$ in general.

We shall remove the indeterminacy of the operation by lifting it to higher torsion groups. First we establish notation. X will always denote an E_∞-space [**9**] and Y will denote an arbitrary space, considered as a subspace of QY via the natural inclusion. We write $K_*(Y; r)$ for $K_0(Y; Z_{p^r}) \oplus K_1(Y; Z_{p^r})$; in particular K-theory is Z_2-graded and we write $|x|$ for the mod 2 degree of x. There are evident natural maps

$$p_*^s \colon K_\alpha(Y; r) \to K_\alpha(Y; r+s) \quad \text{if } s \geq 1,$$
$$\pi \colon K_\alpha(Y; r) \to K_\alpha(Y; t) \quad \text{if } 1 \leq t \leq r,$$

and

$$\beta_r \colon K_\alpha(Y; r) \to K_{\alpha-1}(Y; r).$$

THEOREM 1. *For each $r \geq 2$ and $\alpha \in Z_2$ there is an operation*

$$Q \colon K_\alpha(X; r) \to K_\alpha(X; r-1)$$

with the following properties, where x, $y \in K_*(X; r)$.

(i) *Q is natural with respect to E_∞-maps.*

(ii) $$Q(x+y) = \begin{cases} Qx + Qy - \pi\left[\sum_{i=1}^{p-1} \frac{1}{p}\binom{p}{i} x^i y^{p-i}\right] & \text{if } |x| = |y| = 0, \\ Qx + Qy & \text{if } |x| = |y| = 1. \end{cases}$$

(iii) *$Q\phi = 0$, where $\phi \in K_0(X; r)$ is the identity element.*

(iv) $$Q(xy) = \begin{cases} Qx \cdot \pi(y^p) + \pi(x^p) \cdot Qy + p(Qx)(Qy) & \text{if } |x| = |y| = 0, \\ Qx \cdot \pi(y^p) + p(Qx)(Qy) & \text{if } |x| = 1, |y| = 0, \\ (Qx)(Qy) & \text{if } |x| = |y| = 1. \end{cases}$$

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