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ON PUISEUX SERIES WHOSE CURVES PASS THROUGH AN INFINITY OF ALGEBRAIC LATTICE POINTS

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1. Introduction. Runge [4] proved that certain binary Diophantine equations have only finitely many solutions. Here we give an argument concerning lattice points represented by Puiseux series which proves Runge's Theorem and permits a generalization which shows that there are only finitely many solutions in integers—subject to suitable restrictions—of an algebraic number field. As in the case of Runge's Theorem upper bounds for the absolute value of each solution can be computed, by the methods of the proof.

Let

$$F(x,y)=\sum_{i=0}^{d_1}\sum_{j=0}^{d_2}a_{ij}x^iy^j\in \mathbf{C}[x,y]$$

be of degree d_1 and d_2 in x and y, respectively. Let $\lambda>0$. We define the λ-*leading part*, $F_\lambda(x,y)$, of $F(x,y)$ to be the polynomial consisting of the sum of all terms $a_{ij}x^iy^j$ of $F(x,y)$ for which $i+\lambda j$ is maximal, for that fixed value of λ. We define the *leading part*, $\tilde{F}(x,y)$, of $F(x,y)$ to be the sum of all such terms as λ varies.

We say that an irreducible polynomial

$$F(x,y)\in \mathbf{Z}[x,y]$$

satisfies *Runge's Condition* unless there exists a λ so that $\tilde{F}=F_\lambda$ is a constant multiple of a power of an irreducible polynomial.

Runge's Theorem can now be conveniently formulated: *If $F(x,y)$ satisfies Runge's Condition, then the Diophantine equation $F(x,y)=0$ has only finitely many solutions $(x,y)\in\mathbf{Z}^2$.*

Let L denote an algebraic number field of degree t. Let the conjugates of $\theta\in L$ be denoted by $\theta^{(1)}=\theta,\ \theta^{(2)},\theta^{(3)},\ldots,\theta^{(t)}$, and let

$$\overline{|\theta|}=\max_{1\le\tau\le t}\left|\theta^{(\tau)}\right|.$$

Denote the ring of algebraic integers in L by $\mathcal{O}_L$. We say that $(x,y)\in\mathcal{O}_L^2$ is an *L-lattice point.* We consider certain analytic functions $y=f(x)$, of a complex

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