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EXPONENTIAL GROWTH OF THE l-RANK OF THE CLASS GROUP OF THE MAXIMAL REAL SUBFIELD OF CYCLOTOMIC FIELDS

GARY CORNELL

Introduction. Let K_m be the mth cyclotomic field, so $K_m = Q(e^{2\pi i/m})$ with m not congruent to 2 modulo 4. The maximal real subfield of K_m, K_m^+ and its class number h_m^+ (sometimes called the real class number and sometimes the second factor) is of interest not only to number theorists but to mathematicians in other fields as well. For example, the power of two dividing h_m^+ is tied to classifying certain actions of groups on spheres. (Lang's survey article [7] gives a number of references for this.) However, the real class number is difficult to compute and general information on its size has been hard to come by.

Class Field Theory says that to understand the class group of a number field it would be enough to construct all the unramified abelian extensions of it. A natural idea is to try to construct unramified abelian extensions of a large field by means of abelian (possibly ramified) extensions of some smaller field whose ramification 'collapses' when translated over to the larger field. This idea stems from two seminal papers of Fröhlich [4, 5] where he defined the (absolute) genus field as the largest unramified abelian extension of a number field obtained by composing it with an absolutely abelian field. Unfortunately, it is easy to see that such absolutely abelian translates will never give properly larger abelian unramified extensions of either a full cyclotomic field or its maximal real subfield. Recently [1, 3] it has been shown that by using a properly chosen subfield of a cyclotomic field and abelian extensions of that field (relative genus theory) information about cyclotomic class numbers can be obtained.

This note sketches results obtained by relative genus theory about the class number of cyclotomic fields. Among other results we show that, contrary to what some have believed, the real class number is often quite large. Kubert [6] had earlier obtained results by totally different methods that parallel some of the results obtained here.

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