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THE POINT OF POINTLESS TOPOLOGY[1]

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Introduction. A celebrated reviewer once described a certain paper (in a phrase which never actually saw publication in *Mathematical Reviews*) as being concerned with the study of "valueless measures on pointless spaces". This article contains nothing about measures, valueless or otherwise; but I hope that by giving a historical survey of the subject known as "pointless topology" (i.e. the study of topology where open-set lattices are taken as the primitive notion) I shall succeed in convincing the reader that it does after all have some point to it. However, it is curious that the point (as I see it) is one which has emerged only relatively recently, after a substantial period during which the theory of pointless spaces has been developed without any very definite goal in view. I am sure there is a moral here; but I am not sure whether it shows that "pointless" abstraction for its own sake is a good thing (because it might one day turn out to be useful) or a bad thing (because it tends to obscure whatever point there might be in a subject). That much I shall leave for the reader to decide.

This article is in the nature of a trailer for my book *Stone spaces* **[35]**, and detailed proofs of (almost) all the results stated here will be found in the book (together with a much fuller bibliography than can be accommodated in this article). However, I should make it plain that I do not claim personal credit for more than a small proportion of these results, and that my own understanding of the nature of pointless topology has been enriched by my contacts with a number of other mathematicians, amongst whom I should particularly mention Bernhard Banaschewski, Michael Fourman, Martin Hyland, John Isbell, André Joyal and Myles Tierney. I should also mention the work of Bill Lawvere, particularly as reported in **[41]**, on the nature of continuous variation and the conceptual relation between constant and variable quantities, which has had a profound influence on the developments which I wish to describe; but such questions as these will not be explicitly considered in the present article.

1. Lattices and spaces. It is well known that Hausdorff **[21]** was the first mathematician to take the notion of open set (or neighbourhood) as primitive in the study of continuity properties in abstract spaces. (As Fingerman **[14]** has

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