SCHRÖDINGER SEMIGROUPS

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ABSTRACT. Let \( H = -\frac{1}{2}\Delta + V \) be a general Schrödinger operator on \( \mathbb{R}^n \) (\( n \geq 1 \)), where \( \Delta \) is the Laplace differential operator and \( V \) is a potential function on which we assume minimal hypotheses of growth and regularity, and in particular allow \( V \) which are unbounded below. We give a general survey of the properties of \( e^{-tH} \), \( t > 0 \), and related mappings given in terms of solutions of initial value problems for the differential equation \( du/dt + Hu = 0 \). Among the subjects treated are \( L^p \)-properties of these maps, existence of continuous integral kernels for them, and regularity properties of eigenfunctions, including Harnack's inequality.

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