BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 7, Number 2, September 1982

EXOTIC CLASSES FOR MEASURED FOLIATIONS

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A measured foliation (F, μ) is a C^2-foliation F on a smooth manifold M and a transverse invariant measure μ for F [14]. Inspired by the foliation index theorem of Connes [4, 5], we study the result of integrating *normal data* to F over the leaf space M/F. This produces new secondary-type exotic classes for measured foliations [7]. These classes have applications to SL_q-foliations, to the study of groups of volume-preserving diffeomorphisms, and also are useful for relating the geometry of F to the values of the usual secondary classes [8, 9].

THEOREM 1. *Let (F, μ) be a measured foliation of codimension q on M. If either M is closed and orientable, or μ is absolutely continuous (so it is represented by a closed form $d\mu$), then there is a well-defined characteristic map*

$$\chi_\mu\colon H^*(\mathrm{gl}_q, O_q) \longrightarrow H^{*+q}(M).$$

We call the image of χ_μ the μ-classes of (F, μ).

For M^m compact and $y_I \in H^n(\mathrm{gl}_q, O_q)$, the class $\chi_\mu(y_I)$ is defined as the geometric current in $H_{m-n-q}(M)$ obtained by integrating over the leaf space of F, via μ, the leaf classes corresponding to y_I. Duality then produces the invariant in $H^{n+q}(M)$. If $d\mu$ is a closed form representing μ, then a cocycle representing $\chi_\mu(y_I)$ is $\Delta(y_I) \cdot d\mu$, where $\Delta\colon WO_q \longrightarrow A^\cdot(M)$ is the secondary map for F, [2, 10]. Complete details and properties of χ_μ are described in [7].

The values of the μ-classes depend on the measure μ and the dynamical behavior of F in a neighborhood of the support of μ. It is conjectured that subexponential growth of the leaves of F implies the μ-classes vanish; this can be shown in some cases. Examples can be constructed for which all of the μ-classes are nontrivial.

The canonical measure associated to an SL_q-foliation (F, ω)—where ω is a transverse invariant volume form—defines a characteristic map $\chi_\omega\colon H^*(\mathrm{sl}_q, SO_q) \longrightarrow H^{*+q}(M)$, and these come from universal classes for the Haefliger classifying space $B\Gamma_{SL_q}$. There are additional μ-classes for measured foliations with framed normal bundles, and corresponding universal classes for $B\overline{\Gamma}_{SL_q}$, the homotopy fiber of $B\Gamma_{SL_q} \longrightarrow BSL_q$.

Received by the editors February 23, 1982.
1980 *Mathematics Subject Classification*. Primary 57R30; Secondary 57R20, 28D15.

0273-0979/82/0000-0449/$01.75