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THE RADICAL IN A FINITELY GENERATED P.I. ALGEBRA

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Let R be an associative ring over a commutative ring Λ, $p\{X_1, \ldots, X_e\}$ a polynomial on the free noncommuting variables $X_1, \ldots, X_e$, with coefficients in Λ where one of its coefficient is $+1$. We say that R is a P.I. (polynomial identity) ring satisfying $p\{X_1, \ldots, X_e\}$ if $p(r_1, \ldots, r_e) = 0$ for all $r_1, \ldots, r_e$ in R.

We have the following

THEOREM A. *Let $R = \Lambda\{x_1, \ldots, x_k\}$ be a p.i. ring, where Λ is a noetherian subring of the center $Z(R)$ of R. Then, $N(R)$, the nil radical of R, is nilpotent.*

Recall that $N(R) = \bigcap_P P$ where the intersection runs on all prime ideals of R.

We obtain, as a corollary, by taking Λ to be a field, the following theorem, answering affirmatively the open problem which is posed in **[Pr**, p. 186**]**.

THEOREM B. *Let R be a finitely generated P.I. algebra over a field F. Then, $J(R)$, the Jacobson radical of R, is nilpotent.*

This result, in turn, has the following important consequence.

THEOREM C. *Let $R = F\{x_1, \ldots, x_k\}$ be a finitely generated P.I. algebra over the field F. Then, R is a subquotient of some $n \times n$ matrix ring $M_n(K)$ where K is a commutative F-algebra. Equivalently, there exists an n such that R is a homomorphic image of $G(n, t)$ the ring of t, $n \times n$ generic matrices.*

Kemer, in **[K]**, announced a proof of Theorem B with the additional assumption that $\operatorname{char}(F) = 0$. His proof relies on a result of Razmyslov **[Ra**, Theorem 3**]** and on certain arguments related to the connection between P.I. ring theory and the theory of representation of the symmetric group S_n over F, char $F = 0$. Both results rely heavily on the assumption that char $F = 0$, so they do not seem to generalize directly to arbitrary F.

The previously best known results concerning Theorem A are in **[Ra**, Theorems 1, 3, **Sc**, Theorem 2**]**.

The proof of Theorem C is a straightforward application of Theorem B and a theorem of J. Lewin **[Le**, Theorem 10**]**.

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