

THE JACOBIAN CONJECTURE:
REDUCTION OF DEGREE
AND FORMAL EXPANSION OF THE INVERSE

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INTRODUCTION

A mapping $F: \mathbb{C}^n \rightarrow \mathbb{C}^n$, $F(X) = (F_1(X), \dots, F_n(X))$, is a *polynomial mapping* if each F_i is a polynomial. How do we recognize when such an F is invertible? The question is unambiguous since, once F is bijective, its set theoretic inverse is automatically polynomial (see Theorem 2.1). When F is

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