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PROPER HOLOMORPHIC MAPPINGS EXTEND SMOOTHLY TO THE BOUNDARY

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Biholomorphic mappings between smooth bounded domains in $\mathbf{C}^n$ are known to extend smoothly to the boundary in a wide variety of cases [7, 5, 1]. Much less is known about the boundary behavior of proper holomorphic mappings. In this communication, we sketch the proof of

THEOREM 1. *Suppose $f\colon D_1 \longrightarrow D_2$ is a proper holomorphic mapping between smooth bounded pseudoconvex domains contained in $\mathbf{C}^n$. If the Bergman projection associated to D_1 maps $C^\infty(\overline{D}_1)$ into $C^\infty(\overline{D}_1)$, then f extends smoothly to $\overline{D}_1$.*

Kohn has proved that the Bergman projection associated to a smooth bounded domain D maps $C^\infty(\overline{D})$ into $C^\infty(\overline{D})$ when D is strictly pseudoconvex [11], and more generally, when the boundary of D satisfies certain geometric conditions [12]. Diederich and Fornaess [6] have shown that these conditions are satisfied when the boundary of D is real analytic and pseudoconvex.

REMARKS. (A) Our proof of Theorem 1 uses arguments similar to those used in [2] where it was assumed that the Bergman projection preserved the space of functions which are real analytic up to the boundary. The additional complications encountered in the present work stem from the fact that the ring of germs of smooth functions is not a unique factorization domain.

(B) K. Diederich and J. E. Fornaess have informed us that they also have obtained a proof of Theorem 1 [8].

SKETCH OF THE PROOF OF THEOREM 1. A complete proof of this theorem will appear in [4]. In [3], it is shown that under the hypotheses of Theorem 1, the Jacobian determinant of f, $u = \operatorname{Det}[f']$, extends smoothly to $\overline{D}_1$ and uf^α extends smoothly to $\overline{D}_1$ for each multi-index α. Hence, we are faced with a division problem: to show that u divides uf in $C^\infty(\overline{D}_1)$, given that u and uf^α are in $C^\infty(\overline{D}_1)$ for each α. A necessary first step in attempting to solve this division problem is

LEMMA 1. *Under the hypotheses of Theorem* 1, *the Jacobian* $u = \operatorname{Det}[f']$ *vanishes to at most finite order at each boundary point of* D_1.

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