

## SMOOTH EXTENDABILITY OF PROPER HOLOMORPHIC MAPPINGS

BY KLAS DIEDERICH AND JOHN ERIK FORNAESS

In [9] Ch. Fefferman proved that any biholomorphic mapping  $f: \Omega_1 \rightarrow \Omega_2$  between strictly pseudoconvex  $C^\infty$ -smooth domains in  $\mathbf{C}^n$  extends smoothly to the boundary. Subsequently, the proof of this result has been simplified considerably by S. Webster [14, 15], E. Ligočka [12], St. Bell [1, 2]. And it was St. Bell who realized the importance of the following regularity condition of the Bergman projection for the proof of such extendability results:

**DEFINITION.** A domain  $\Omega \subset \subset \mathbf{C}^n$  is said to satisfy condition  $R$  for its Bergman projection operator  $P$  if for any positive integer  $s$  there is an integer  $N$  such that  $P$  is a bounded linear operator from  $W_0^{s+N}(\Omega)$  to  $H^s(\Omega)$ .

(Here  $W_0^s(\Omega)$  denotes as usual the closure of  $C_0^\infty(\Omega)$  in the Sobolev  $s$ -Norm  $\|\cdot\|_s$  with respect to the volume Lebesgue-measure on  $\Omega$  and  $H^s(\Omega)$  is the space of holomorphic functions on  $\Omega$  with finite  $\|\cdot\|_s$ -norm.)

Since condition  $R$  is a consequence of subelliptic estimates for the  $\bar{\partial}$ -Neumann problem, it is known to be satisfied for instance in the following cases:

- (1)  $\Omega$  strictly pseudoconvex,  $C^\infty$ -smooth (J. J. Kohn [10]);
- (2)  $\Omega$  pseudoconvex,  $C^\omega$ -smooth (J. J. Kohn [11], K. Diederich, J. E. Fornaess [6]).

The new methods allowed to generalize Fefferman's result. It is now known that a biholomorphic mapping  $f: \Omega_1 \rightarrow \Omega_2$  extends smoothly up to the boundary if  $\Omega_1$  and  $\Omega_2$  are  $C^\infty$ -smooth and, in addition, both satisfy condition  $R$  [1] or both are pseudoconvex and at least one satisfies condition  $R$  [2].

The result which we wish to announce deals with the case of proper holomorphic mappings and is contained in the

**THEOREM.** *Let  $\Omega_1, \Omega_2 \subset \subset \mathbf{C}^n$  be  $C^\infty$ -smooth pseudoconvex domains and suppose that  $\Omega_1$  satisfies condition  $R$ . Then any proper holomorphic mapping  $f: \Omega_1 \rightarrow \Omega_2$  extends smoothly up to the boundary.*

For unbranched mappings  $f$  this result is contained in K. Diederich and J. E. Fornaess [7]. This also includes the case of  $\Omega_1, \Omega_2$  being strictly pseudoconvex and  $f$  proper holomorphic since any such  $f$  is necessarily unbranched, S. Pincuk [13]. Under different, more restrictive assumptions on  $\Omega_1$  and  $\Omega_2$  the result

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