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UNIPOTENT AND PROUNIPOTENT GROUPS: COHOMOLOGY AND PRESENTATIONS

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A pro-affine algebraic group G, over the field k (which we always take to be algebraically closed of characteristic zero) is an inverse limit of affine algebraic groups [**3**]. If the algebraic groups in the inverse system are unipotent, we call G *prounipotent.* Pro-affine algebraic groups arise naturally in the theory of finite-dimensional k-representations of discrete and analytic groups [**3**, **4**, **9**] and prounipotent groups arise naturally as the prounipotent radicals of pro-affine groups. Our interest in prounipotents is motivated by possible applications to finite-dimensional representation theory.

The extension of the category of unipotent groups to that of prounipotents makes possible "combinatorial group theory" (free groups and presentations):

If X is a set, there is a prounipotent group $F(X)$ containing X such that for every prounipotent group H and function $f: X \longrightarrow H$ with $\mathrm{Card}\{X - f^{-1}(L)\}$ finite for every closed subgroup L of finite codimension in H there is a unique homomorphism $\bar{f}: F(X) \longrightarrow H$ extending f [**5**, 2.1]. Every prounipotent group G is a homomorphic image of a free prounipotent group F so there is an exact sequence (*) $1 \longrightarrow R \longrightarrow F \longrightarrow G \longrightarrow 1$. We can choose (*) with $R \subseteq (F, F)$ and in this case we call (*) a *proper presentation* of G. If $F = F(X)$ in (*), we call X generators for G and we call generators of R, as a prounipotent normal subgroup of F, relations for G.

As for pro-p groups [**11**], the numbers of generators and relations for G have a cohomological interpretation. Cohomology here is in the category of polynomial representations as in [**2**]. There is a unique simple in this category (the one-dimensional trivial module k) so cohomological dimension is defined as $\mathrm{cd}(G) = \inf\{i \mid H^n(G, k) = 0, n > i\}$.

THEOREM 1 [**5**, 2.8 AND 2.9]. *The following are equivalent for prounipotent G*:

(a) *G is free,*

(b) *G is a projective group in the category of prounipotent groups,*

(c) $\mathrm{cd}(G) \leqslant 1$.

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