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A SOLUTION TO A PROBLEM OF J. R. RINGROSE[1]

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We announce a solution to a multiplicity problem for nests posed by J. R. Ringrose approximately twenty years ago. This also answers a question posed by R. V. Kadison and I. M. Singer, and independently by I. Gohberg and M. Krein concerning the invariant subspace lattice of a compact operator. The key to the proof is a result concerning compact perturbations of nest algebras which was recently obtained by Niels Andersen in his doctoral dissertation. The complete proof of the general result as well as of a number of related results will appear elsewhere. A proof for the special case which answers Ringrose's original question is included herein.

Let H be infinite dimensional separable Hilbert space. A *nest* $\mathcal{N}$ is a family of closed subspaces of H linearly ordered by inclusion. $\mathcal{N}$ is *complete* if it contains $\{0\}$ and H and contains the intersection and the join (closed linear span) of each subfamily. The corresponding *nest algebra* alg $\mathcal{N}$ is the algebra of all operators in $L(H)$ which leave every member of $\mathcal{N}$ invariant. The *core* $\mathcal{C}_{\mathcal{N}}$ is the von Neumann algebra generated by the projections on the members of $\mathcal{N}$, and the *diagonal* $\mathcal{D}_{\mathcal{N}}$ is the von Neumann algebra (alg $\mathcal{N}$) $\cap$ (alg $\mathcal{N}$)*. $\mathcal{N}$ is *continuous* if no member of $\mathcal{N}$ has an immediate predecessor or immediate successor. Equivalently, $\mathcal{N}$ is continuous if the core $\mathcal{C}_{\mathcal{N}}$ is a nonatomic von Neumann algebra. $\mathcal{N}$ has *multiplicity one* (is multiplicity free) if $\mathcal{D}_{\mathcal{N}}$ is abelian, or equivalently, if $\mathcal{C}_{\mathcal{N}}$ is a m.a.s.a.

J. R. Ringrose posed the following question: Let $\mathcal{N}$ be a multiplicity free nest and $T: H \longrightarrow H$ a bounded invertible operator. Is the image nest $T\mathcal{N} = \{TN: N \in \mathcal{N}\}$ necessarily multiplicity free? Note that $T(\text{alg } \mathcal{N})T^{-1} = \text{alg}(T\mathcal{N})$, so it is natural to say that $T\mathcal{N}$ is the similarity transform of $\mathcal{N}$. Is multiplicity preserved under similarity? We show that the answer is no. It should be noted that a negative answer was conjectured in recent years by several mathematicians including J. Ringrose and W. Arveson.

The following key result is due to N. Andersen [1]. Let $\mathcal{LC}$ denote the compact operators in $L(H)$.

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