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FAST RECURSION FORMULA FOR WEIGHT MULTIPLICITIES[1]

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The purpose of this note is to describe and prove a fast recursion formula for computing multiplicities of weights of finite dimensional representations of simple Lie algebras over **C**.

Until now information about weight multiplicities for all but some special cases [1, 2] has had to be found from the recursion formulas of Freudenthal [3] or Racah [4]. Typically these formulas become too laborious to use for hand computations for ranks $\gtrsim 5$ and dimensions $\gtrsim 100$ and for ranks $\simeq 10$ and dimensions $\simeq 10^4$ on a large computer [5, 6]. With the proposed method the multiplicities can routinely be calculated, even by hand, for dimensions far exceeding these. As an example we present a summary of calculations [7] of all multiplicities in the first sixteen irreducible representations of E_8.

Let $\mathfrak{G}$ be a semisimple Lie algebra over **C** with root system Δ and Weyl group W relative to a Cartan subalgebra $\mathfrak{H}$. Let Δ^+ be the positive roots with respect to some ordering and $\Pi = \{\alpha_1, \ldots, \alpha_l\}$ the set of simple roots. Let Q and P be the root and weight lattices respectively spanning the real vector space $V \subset \mathfrak{H}^*$. If $X \subset P$ we denote by X^{++} the set of dominant elements of X relative to Π.

Let M be an irreducible $\mathfrak{G}$-module with highest weight Λ and weight system Ω. An important feature of the approach is the direct determination of Ω^{++} without computing outside the dominant chamber. Since every W-orbit is represented by one weight $\lambda \in \Omega^{++}$ of the same multiplicity, it suffices to compute such λ's.

The recursion formula for computing the multiplicities is a modification (Proposition 4) of the Freudenthal formula in which the Weyl group has been exploited to collapse it as much as possible. After describing the procedure, we present the E_8 example. Finally the necessary proofs are given.

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