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A TORELLI THEOREM FOR SIMPLY CONNECTED ELLIPTIC SURFACES WITH A SECTION AND $p_g \geqslant 2$

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0. The purpose of this note is to announce some recent results on the period mapping for simply connected elliptic surfaces with a section and $p_g \geqslant 2$. Among other things, we are able to prove the period mapping for such surfaces has degree one, that is to say, it is generically injective. In the terminology of **[C, G]**, this is called the weak global Torelli theorem. In order to make a precise statement of our results, we must introduce some notation.

1. Let V be a surface with at most rational double points, $\overline{V} \xrightarrow{\sigma} V$ its minimal resolution; let $\psi: V \longrightarrow \mathbf{P}_1$ be a proper analytic map, whose generic fibre is a smooth elliptic curve. Let $C_u = \psi^{-1}(u)$, $\overline{\psi} = \psi \circ \sigma$, and $\overline{C}_u = \overline{\psi}^{-1}(u)$. Assume $\exists s \ni s: \mathbf{P}_1 \longrightarrow V$ is a section of $V \xrightarrow{\psi} \mathbf{P}_1$, and that V is smooth along $s(\mathbf{P}_1)$; let $\overline{s}$ be the corresponding section of $\overline{V} \xrightarrow{\overline{\psi}} \mathbf{P}_1$. Now assume $p_g(\overline{V}) = n \geqslant 1$, and $\overline{V}$ is a minimal surface; $\overline{s} \cdot \overline{C}_u = 1$, $\overline{s}^2 = -(n+1)$, $\overline{C}_u^2 = 0$. $V \xrightarrow{\psi} \mathbf{P}_1$ is called an *elliptic fibration* with a section (see **[K]**). We also require that $\sigma(D) =$ a point for all irreducible curves $D \subset \overline{V}$, with $\overline{s} \cdot D = 0 = \overline{C}_u \cdot D$.

Let $H^2(\overline{V}; \mathbf{Z})_0 = (\mathbf{Z}c_1(\overline{C}_u) + \mathbf{Z} \cdot c_1(\overline{s}))^\perp$, where $c_1(D)$ denotes the 1st Chern class of a divisor; this is an orthogonal direct summand of $H^2(\overline{V}; \mathbf{Z})$ and hence must be a unimodular lattice. It is uniquely characterized by the integer n; fix a copy and call it H; set $\Lambda = H \oplus (\mathbf{Z}c + \mathbf{Z}s)$, where $c \cdot H = s \cdot H = 0$, $c^2 = 0$, $s \cdot c = 1$, $s^2 = -(n+1)$.

Let $\varphi: H^2(\overline{V}; \mathbf{Z}) \longrightarrow \Lambda$ be an isomorphism of unimodular lattices $\varphi(c_1(\overline{C}_u)) = c$, $\varphi(c_1(\overline{s})) = s$, $\varphi(c_1(K_{\overline{V}})) = (n-1) \cdot c$ where $K_{\overline{V}}$ is the canonical divisor, and hence $\varphi(H^2(\overline{V}; \mathbf{Z})_0) = H$. Following **[P-S, Šaf]**, we will call φ a marking, and the pair (V, φ) a *marked surface* of type (H, Λ, c, s). Let $L_V \subset H^2(\overline{V}; \mathbf{Z})_0$ be the euclidean sublattice generated by all elements of the form $c_1(D), D > 0$, $\sigma(D) =$ a point; and let $\widetilde{H}_V = (L_V)^\perp$; L_V is negative definite. For any euclidean lattice E, we let $O(E)$ be the orthogonal group of E.[1] This is a linear algebraic group defined over $\mathbf{Q}$; let $O(E)_\mathbf{Z} = \{g \in O(E)_\mathbf{R} \,|gE = E\}$. For any $\alpha \in E$, $\alpha^2 = -2$, let

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[1] The pairing on E may be indefinite.