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RESONANCE FOR QUASILINEAR HYPERBOLIC EQUATION[1]

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In this note we announce results on a nonlinear conservation law with a moving source.

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = g(x - dt, u). \tag{1}$$

The equation is intended to model fluid motions under external effects, either physical or geometrical, such as gas flows through a nozzle, MHD shock tube [**1**, **6**, **9**]. As such, we assume that the flux $f(u)$ is a smooth convex function of the density u, and the source term has the form

$$g(\xi, u) = c(\xi)h(u), \qquad \xi = x - dt,$$

where $c(\xi)$ is a piecewise continuous function and $h(u)$ is a smooth positive function whose first derivative does not change signs. The external effect is assumed to be finite; for simplicity, we suppose also that $c(\xi)$ has compact support.

Our main interest is the behavior of nonlinear waves when the resonance occurs, that is, when the characteristic speed $f(u)$ is close to the speed d of the source. The behavior of nonresonance waves for general systems of conservation laws with source terms has been studied in [**6**]. These waves are dynamically stable. As a first step to understand the resonance effects, we study the interaction of shock waves and rarefaction waves for the conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial t} = 0 \tag{2}$$

[**3**, **5**] and the steady traveling waves with speed d; that is, solutions of

$$\frac{d(f(u) - du)}{d\xi} = c(\xi)h(u), \qquad \xi = x - dt \tag{3}$$

[**6**]. When a *transonic* shock wave $(u_-, u_+), f'(u_-) > d > f'(u_+)$, propagates through a steady traveling wave it accelerates (or decelerates) and therefore is unstable (or stable) if $c(\xi)h'(u)$ is negative (or positive). More interestingly, as a

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